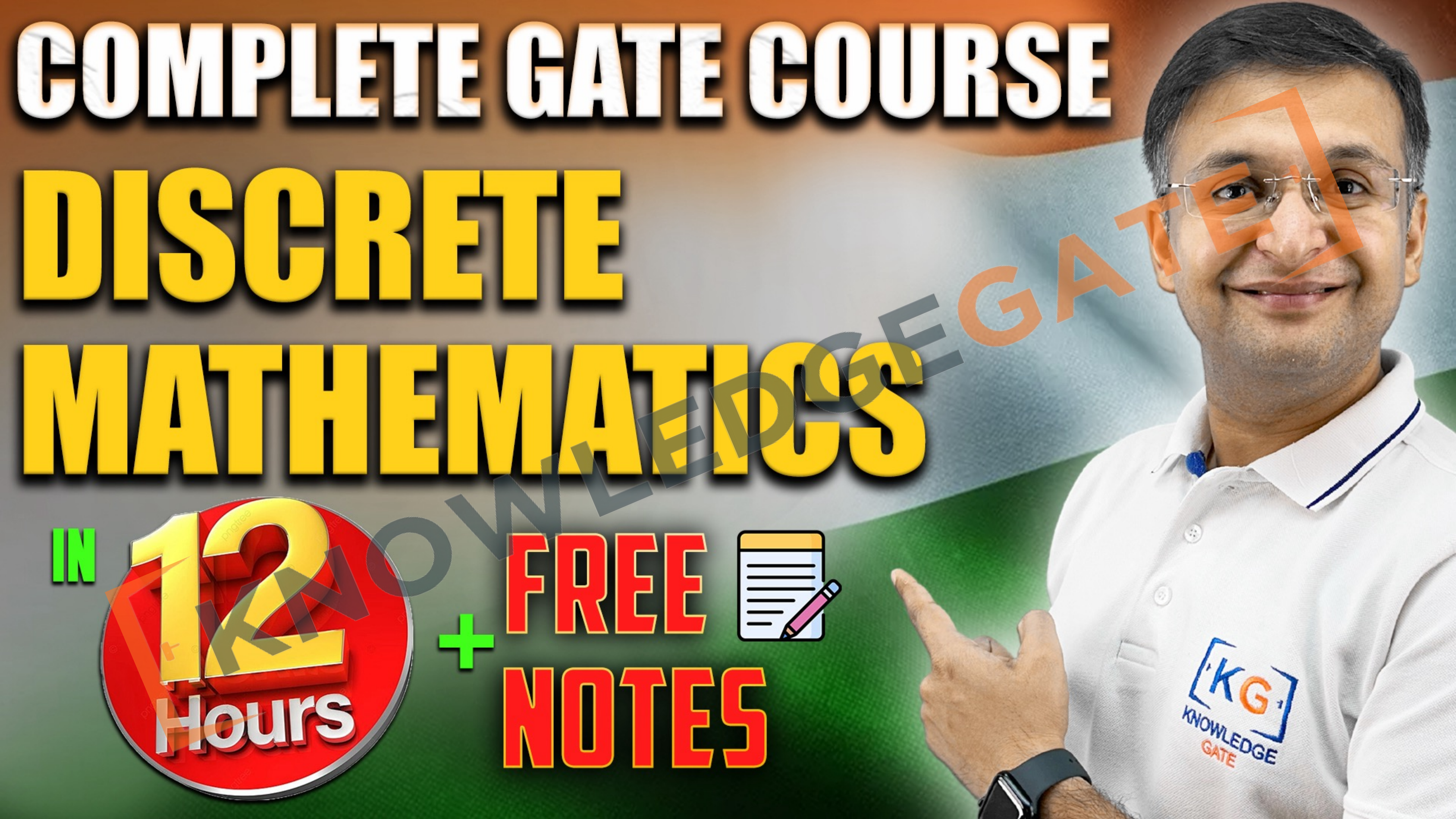
COMPLETE GATE COURSE
DISCRETE MATHEMATICS
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- Discrete mathematics is a core subject of theoretical computer science. It is not a directly application-oriented subject, but it provides tolls and mathematical models, which are applied to different areas in computer science.

- **GATE (7-9 MARKS)**

- Has a good weightage in general all objective and subjective examination.

- Will be asked in Interview for M.Tech, PhD or other government jobs. Not that important in software industry.

Section 1: Engineering Mathematics

Discrete Mathematics: Propositional and first order logic. Sets, relations, functions, partial orders and lattices. Monoids, Groups. Graphs: connectivity, matching, coloring. Combinatorics: counting, recurrence relations, generating functions.

Linear Algebra: Matrices, determinants, system of linear equations, eigenvalues and eigenvectors, LU decomposition.

Calculus: Limits, continuity and differentiability. Maxima and minima. Mean value theorem. Integration.

Probability and Statistics: Random variables. Uniform, normal, exponential, poisson and binomial distributions. Mean, median, mode and standard deviation. Conditional probability and Bayes theorem.

Video chapters

- Chapter-1 (Set Theory):
- Chapter-2 (Relations):
- Chapter-3 (POSET & Lattices):
- Chapter-4 (Functions):
- Chapter-5 (Graph Theory):
- Chapter-6 (Group Theory):
- Chapter-7 (Proposition):

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Chapter-1 (Set Theory)

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What is a SET

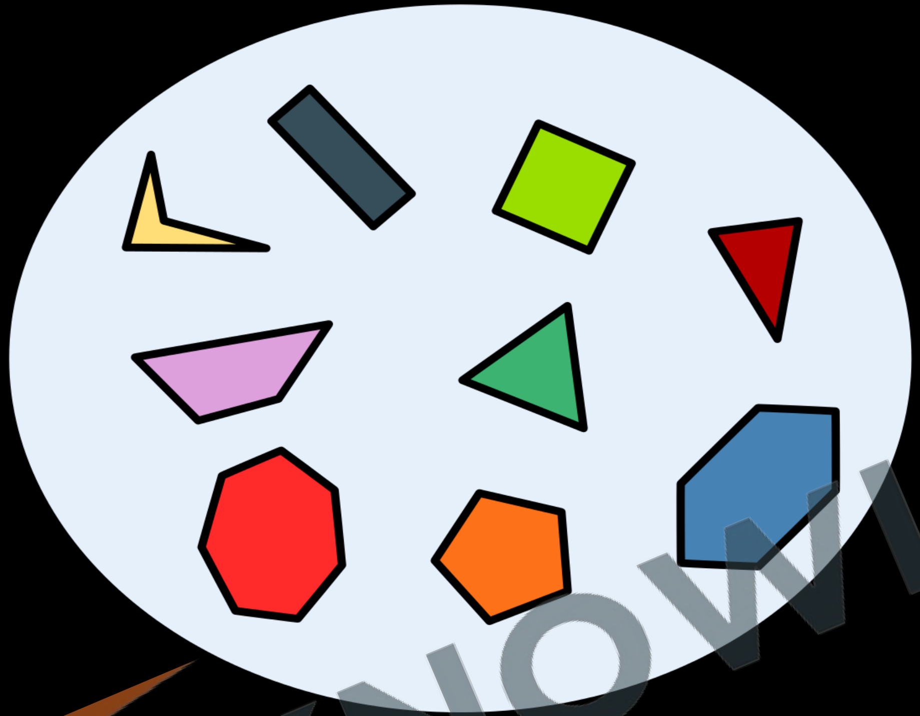
- Set are the fundamental discrete structures on which all the discrete structures are built. Sets are used to group objects together, formally speaking

- "An *unordered ,well-defined*, collection of *distinct* objects (Called elements or members of a set) of same type". Here the type is defined by the one who is defining the set. For e.g.

- A = {0,2,4,6, ---}

- B = {1,3,5, ---}

- C= {x| x ∈ Natural number}



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- A Set is generally denoted usually by capital letter. The objects of a set called the elements, or members of the set.
- A set is said to contain its elements.
- Lower case letters are generally used to denote the elements of the set.

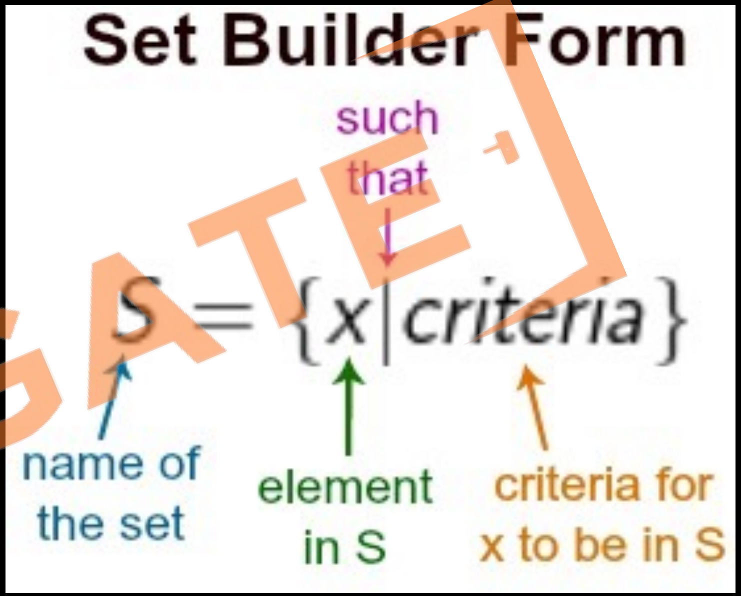
- $x \in A$, means element x is a member of A

- $x \notin A$ means x is not a member of A

Representation of set

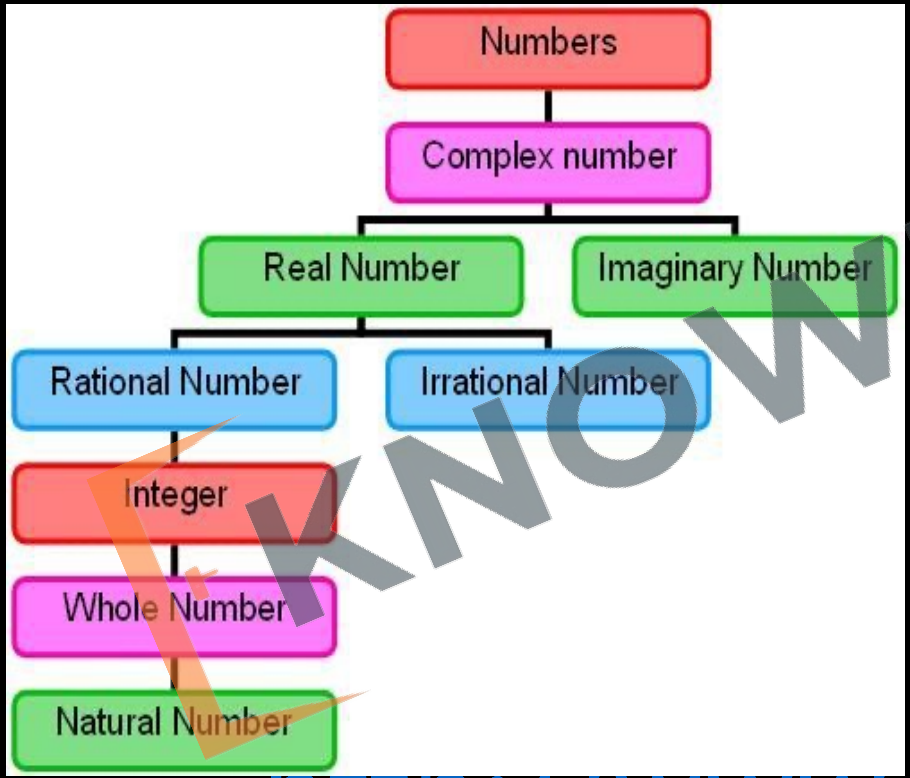
- **Tabular/Roster representation of set** - here a set is defined by actually listing its members. E.g.
- A = {a, e, i, o, u}
- B = {1, 2, 3, 4}
- C = {---- -4, -2, 0, 2, 4, -----}.

- **Set Builder representations of set**- here we specify the property which the elements of the set must satisfy. E.g.

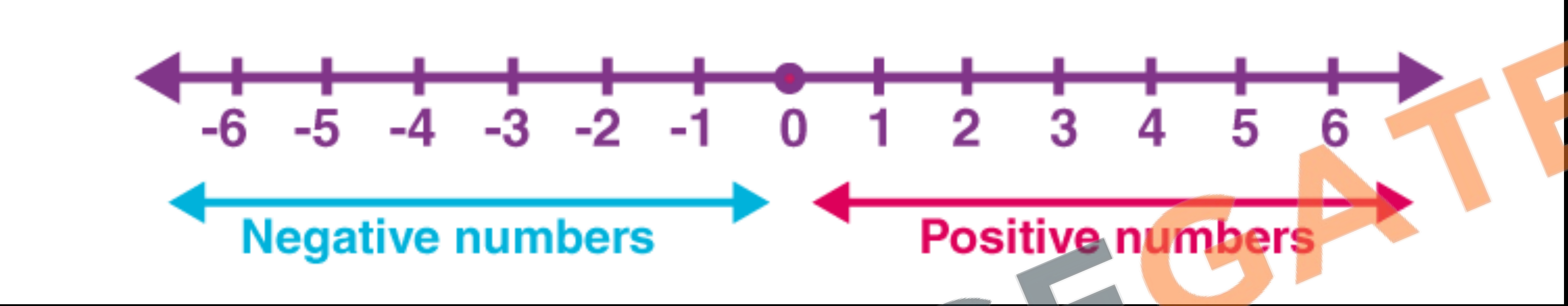


- A = {x| x is an odd positive number less than 10},
- A = {x | x ϵ English alphabet && x is vowel}
- B = {x | x ϵ N && x <5}
- C = {x | x ϵ Z && x%2 = 0 }

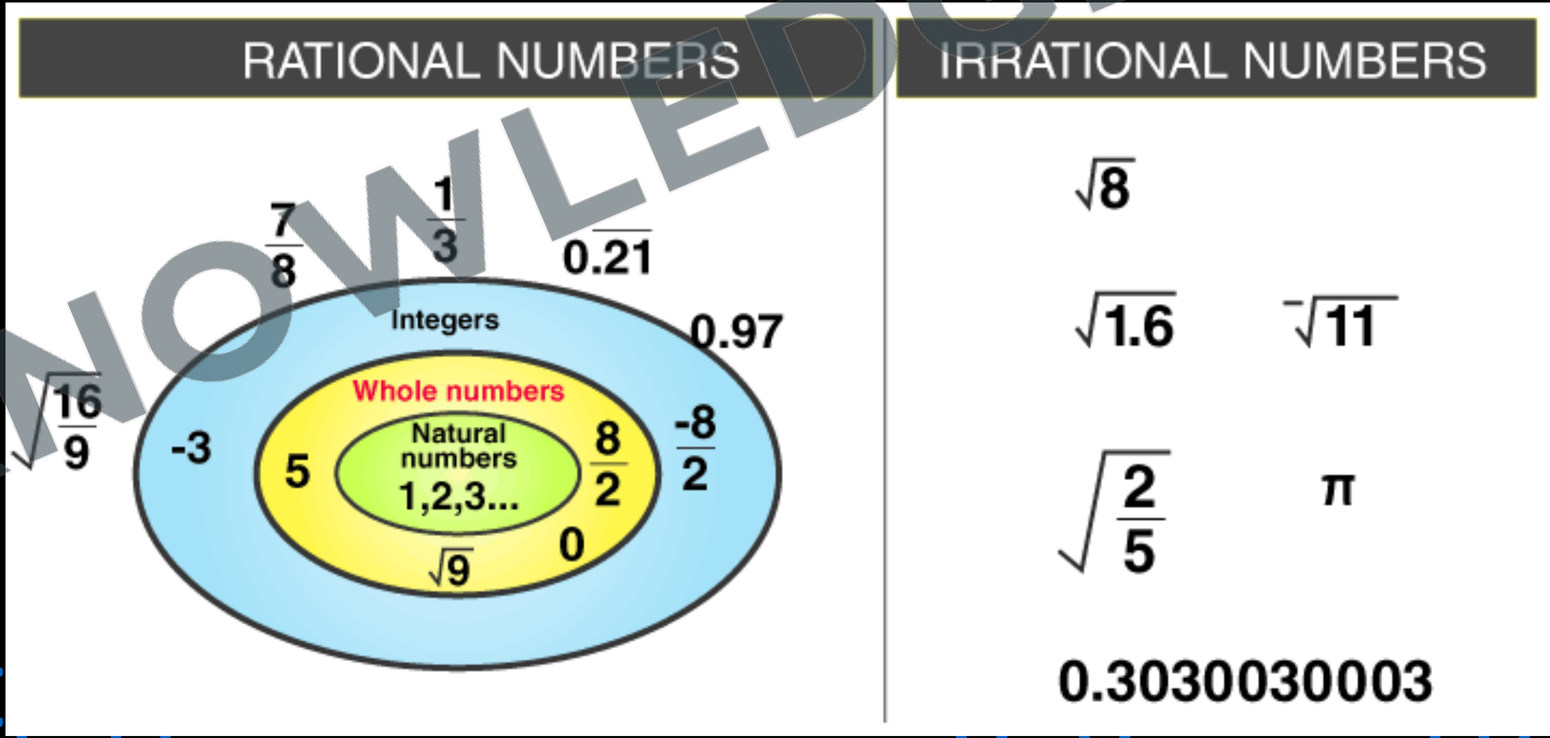
- **<u>Set of all-Natural number(N)</u> -** A natural number is a number that occurs commonly and obviously in nature. The set of natural numbers, can be defined as N = {1,2,3, 4.... ∞}
- **<u>Set of all Whole number(W)</u> -** A whole number is a science expanded natural number. Set of natural number and zero



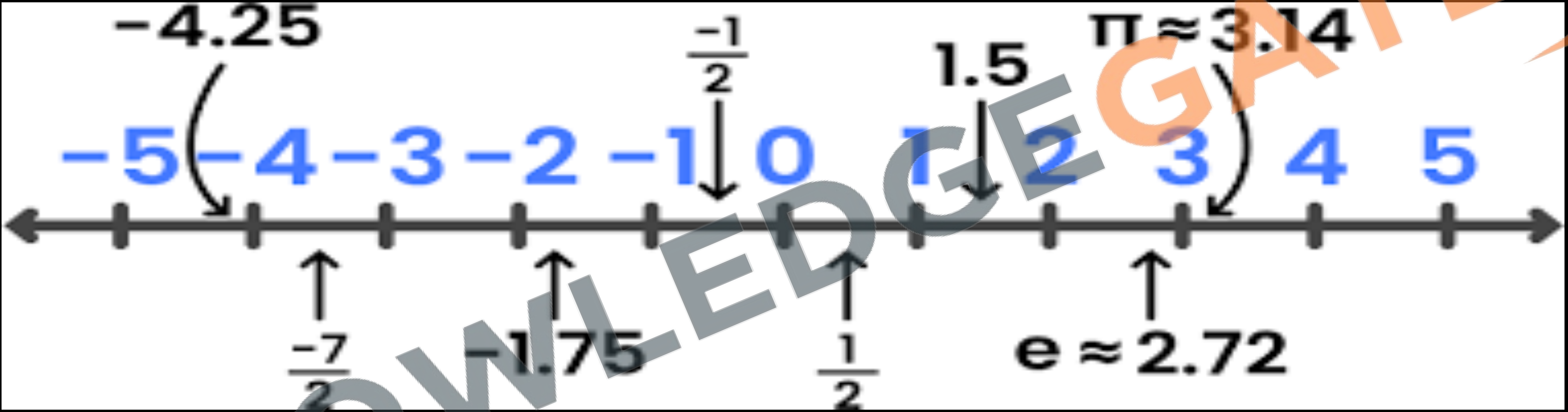
- **Set of all Integer(Z)** - An integer is a number that can be written without a fractional component.



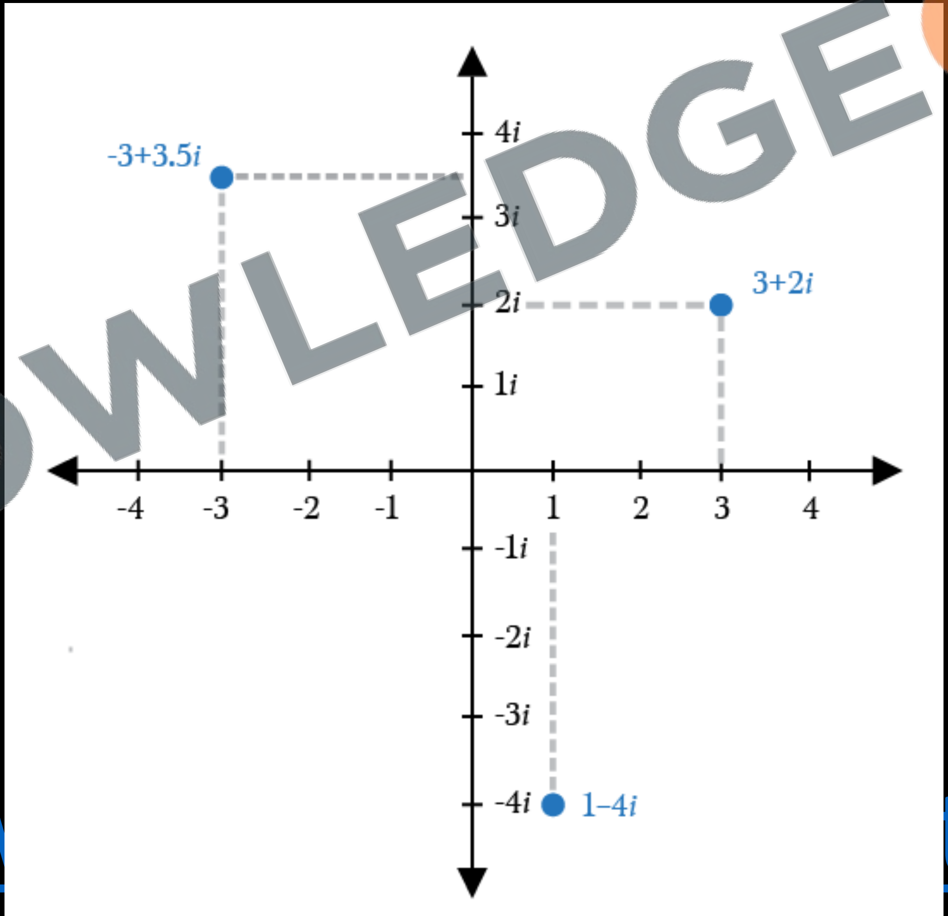
- **<u>Set of all Rational number (Q)</u> -** A rational number is any number that can be expressed as a fraction P/Q of two integers, a numerator P and a non-zero denominator Q.
- **<u>Set of all Irrational number (R-Q or R/Q or P)</u> -** An irrational number is a real number that cannot be expressed as a fraction i.e. as a ratio of integers. Therefore, irrational numbers, when written as decimal numbers, do not terminate, nor do they repeat. E.g. root2.



- **Set of all Real number(R) -** A real number is a value that represents a quantity along a continuous line, containing all of the rational numbers and all of the irrational numbers.



- **Set of all Complex number(C) -** A complex number is a number that can be expressed in the form ‘a + bi’, where ‘a’ and ‘b’ are real numbers and ‘i’ is the imaginary unit, that satisfies the equation $i^2 = -1$. In this expression, ‘a’ is the real part and ‘b’ is the imaginary part of the complex number.



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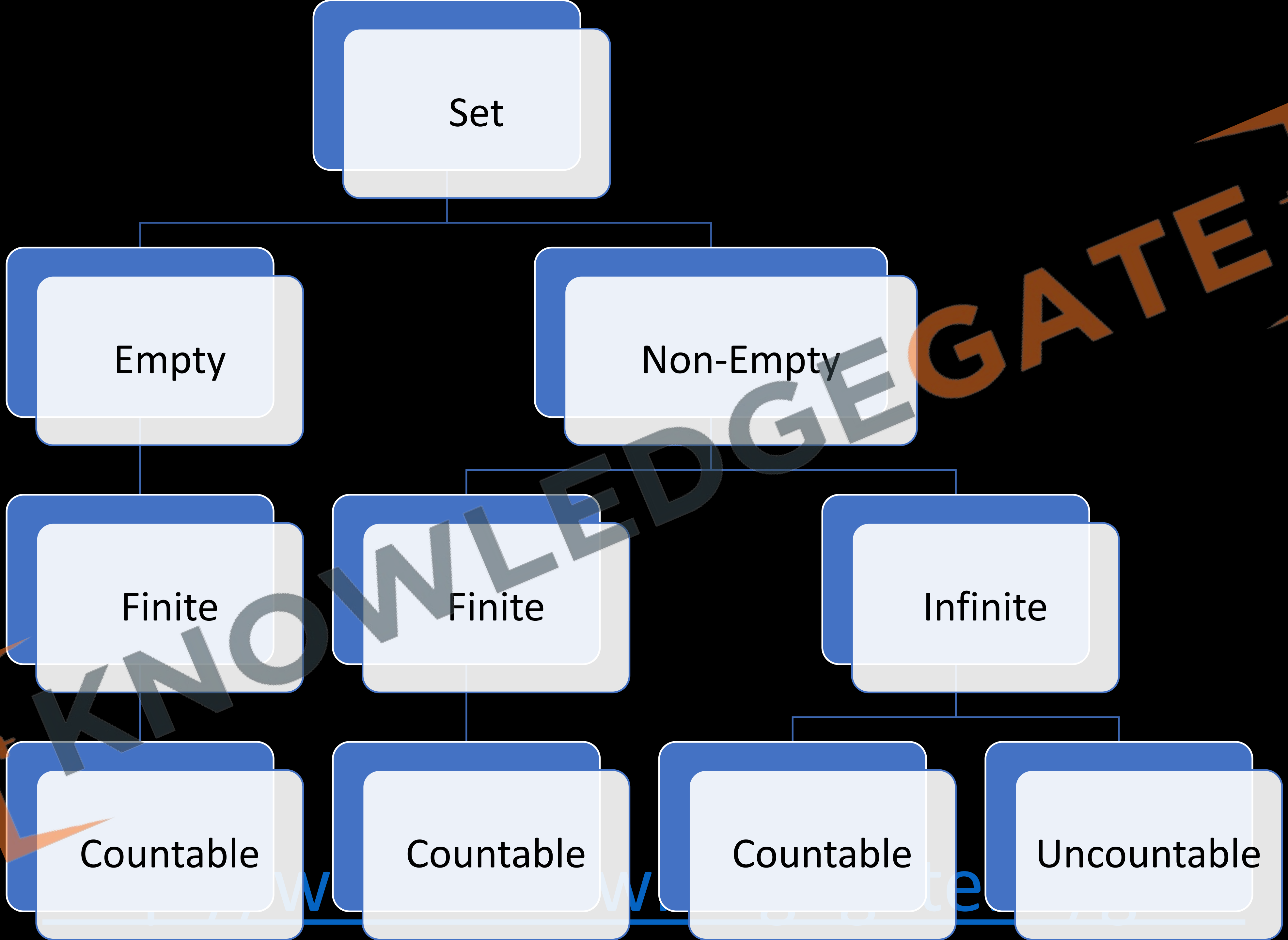
- **Finite set** - If there are exactly *‘n’* elements in *S* where *‘n’* is a nonnegative integer, we say that *S* is a *finite set.*
- *i.e. if a* set contain specific or finite number of elements is called is called finite set. For e.g. A = {1,2,3,4}

- **Cardinality of a set** — It is the number of elements present in a finite Set, denoted like $|A|$.
- For e.g. $A = \{0,2,4,6\}$, $|A| = 4$

- **Infinite set** – A set contain infinite number of elements is called infinite set, if the counting of different elements of the set does not come to an end. For e.g. a set of natural numbers.


Set
Empty
Non-Empty
Finite
Finite
Infinite
Countable
Countable
Countable
Uncountable

- **Countable set** – A set is said to be countable if there can be a one to one mapping between the elements of the set and natural numbers. E.g. Set of stars, N, W, Z, Q.

- **Uncountable set** – A set is said to be uncountable if there cannot be a one to one mapping between the elements of the set and natural numbers. E.g. Set of real numbers.

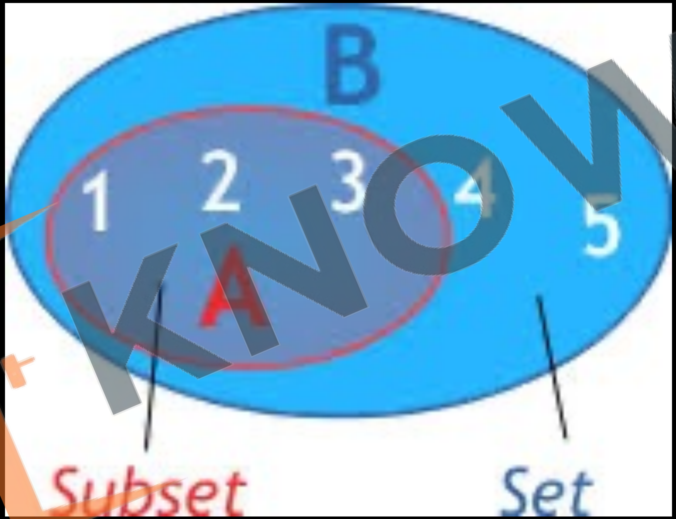
- **Null set / empty set** - Is the unique set having no elements. its size or cardinality is zero i.e. |ϕ| = 0. It is denoted by a symbol ϕ or {}.

- **Universal set** – if all the sets under investigation are subsets of a fixed set, i.e. the set containing all objects under investigation, in Venn diagram it is represented by a rectangle, and it is denoted by U.

- **<u>Subset of a set</u>** – If every element of set A is also an element of set B i.e.
- $\forall x(x \in A \rightarrow x \in B)$, then A is called subset of B and is written as $A \sqsubseteq B$. B is called the superset of A.

- E.g. A = {1,2,3} B = {1,2,3,4,5}

- Note that to show that *A* is not a subset of *B* we need only find one element $x \in A$ with $x \notin B$. To show that $A \sqsubseteq B$, show that if $x \in A$, then $x \in B$.



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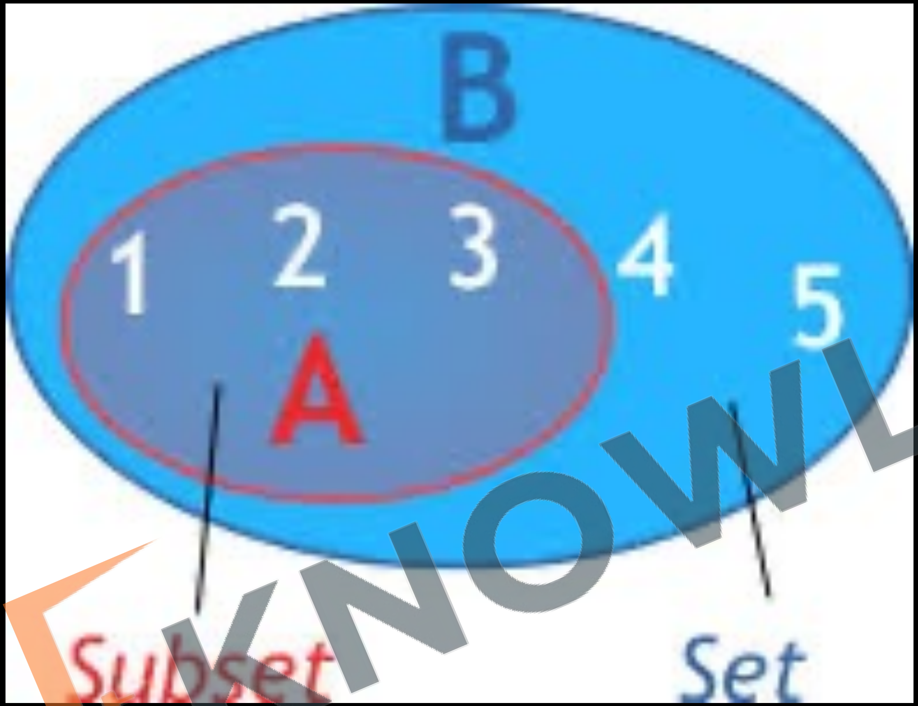
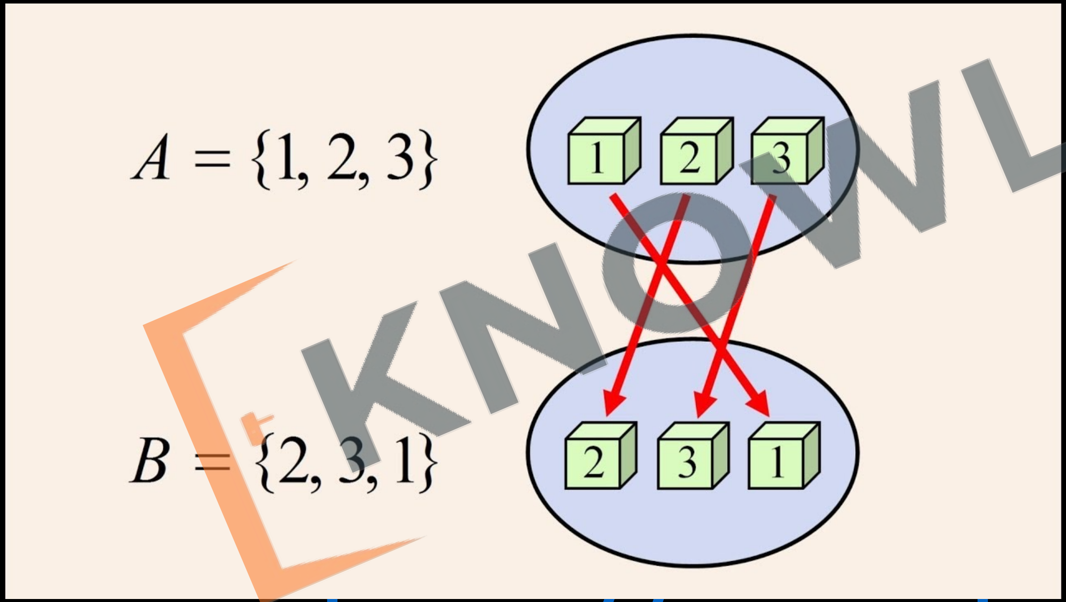
- $\phi \sqsubseteq A$, Empty Set ϕ is a subset for every set
- $A \sqsubseteq U$, Every Set is a subset of Universal set U
- $A \sqsubseteq A$, Every Set is a subset of itself.

- **Proper subset** – if A is a subset of B and A ≠ B, then A is said to be a proper subset of B, i.e. there is at least one element in B which is not in A. denoted as $A \subset B$.



- **Equality of sets** – If two sets A and B have the same element and therefore every element of A also belong to B and every element of B also belong to A, then the set A and B are said to be equal and written as A=B.
- if $A \sqsubseteq B$ and $B \sqsubseteq A$, then A=B
- $\forall x(x \in A \leftrightarrow x \in B)$



- **Power set** – let A be any set, then the set of all subsets of A is called power set of A and it is denoted by P(A) or 2^A.

- If A= {1,2,3}, then P(A) = {ϕ, {1}, {2}, {3}, {1,2}, {2,3}, {1,3}, {1,2,3}}

- Cardinality of the power set of A is n, $|P(A)| = 2^n$

Q For any Set A, which of the following are true?

1) $\phi \in A$

2) $\phi \sqsubseteq A$

3) $\phi \in 2^A$

4) $\phi \sqsubseteq 2^A$

5) $A \in 2^A$

6) $A \sqsubseteq 2^A$

Q If ϕ is an empty set. Then | P(P(P(ϕ))) | =_______?

a) 1 **b)** 2 **c)** 4 **d)** none of above

Q The cardinality of the power set of {0, 1, 2 . . ., 10} is __________.

Q For a set A, the power set of A is denoted by 2^A. If A = {5, {6}, {7}}, which of the following options are True.

I) $\phi \in 2^A$ **II)** $\phi \sqsubseteq 2^A$ **III)** $\{5, \{6\}\} \in 2^A$ **IV)** $\{5, \{6\}\} \sqsubseteq 2^A$

(A) I and III only

(B) II and III only

(C) I, II and III only

(D) I, II and IV only

Q Let P(S) denotes the power set of set S. Which of the following is always true?

a) $P(P(S)) = P(S)$

(b) $P(S) \cap P(P(S)) = \{\phi\}$

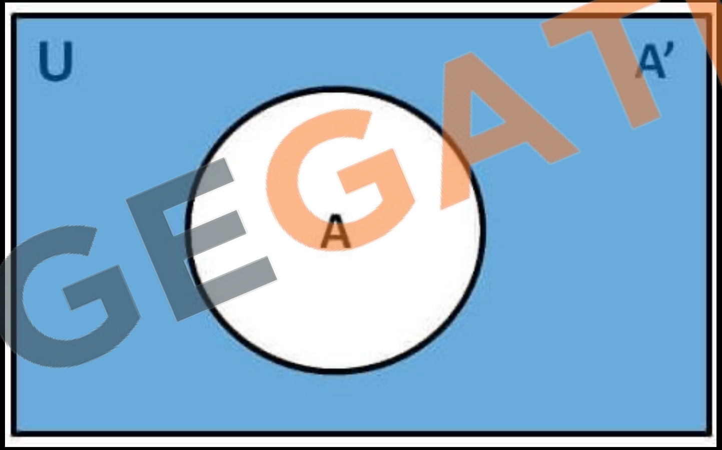
(c) $P(S) \cap S = P(S)$

(d) $S \notin P(S)$

Q The number of elements in the power set P(S) of the set S= {{Ø},1, {2,3}} is

Operation on sets

- **Complement of set** – Set of all x such that $x \notin A$, but $x \in U$.
- $A^c = \{x \mid x \notin A \ \& \ x \in U\}$

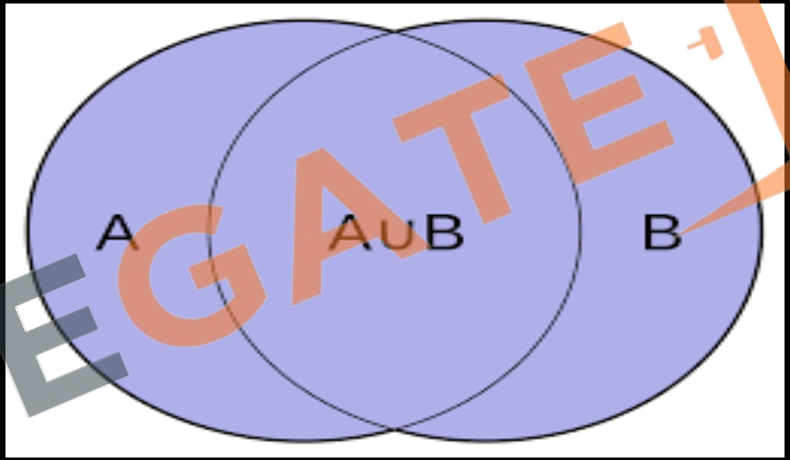


$U = \{1, 2, 3, 4, 5, 6\}$

$A = \{2, 3, 6\}$

$A^c = \{\quad\}$

- **Union of sets** – Union of two sets A and B is a set of all those elements which either belong to A or B or both, it is denoted by A U B.
- A U B = {x| x ϵ A or x ϵ B}



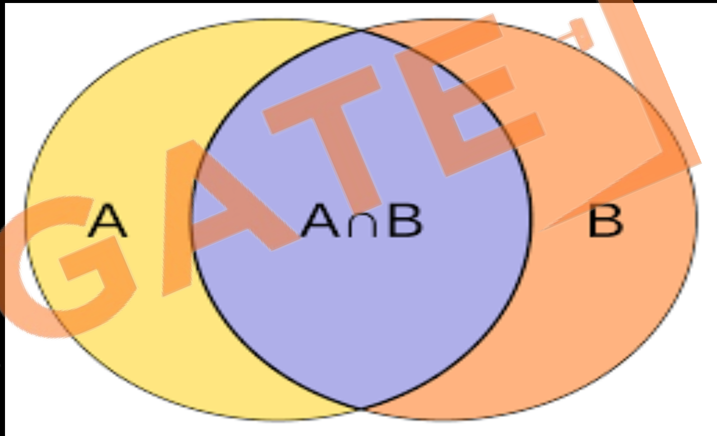
A = {1, 2, 3, 4}

B = {3, 4, 5, 6}

A U B = { }

|A|+|B|=| A U B | ?

- **Intersection of sets** - Intersection of two sets A and B is a set of all those elements which belong to both A and B, and is denoted by $A \cap B$.
- $A \cap B = \{x \mid x \in A \text{ and } x \in B\}$

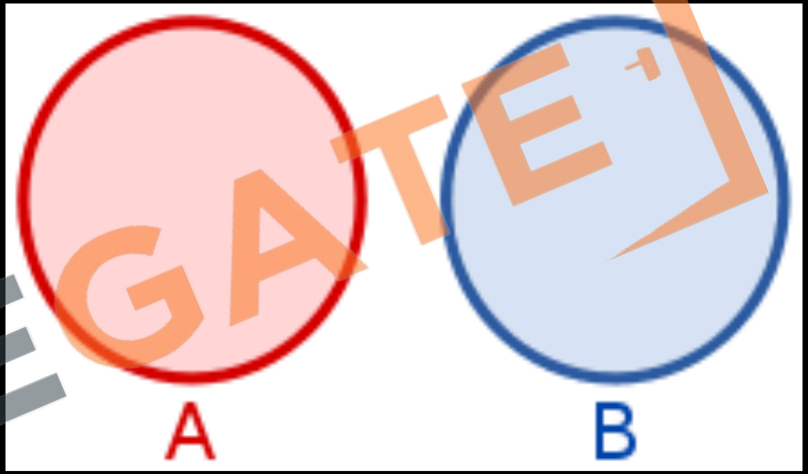


$A = \{1, 2, 3, 4\}$

$B = \{3, 4, 5, 6\}$

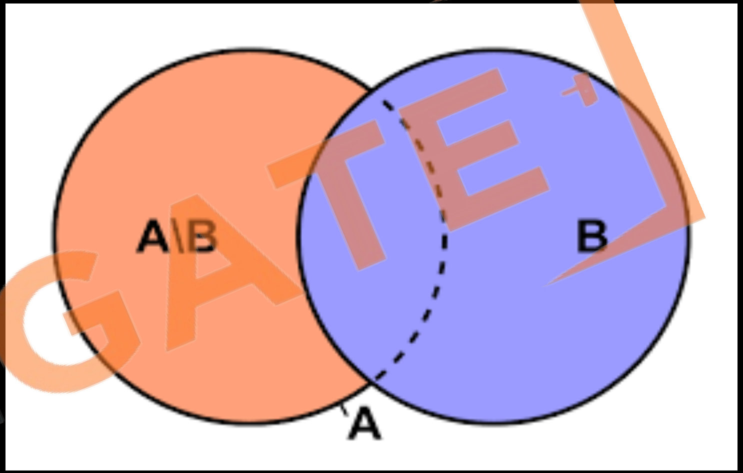
$A \cap B = \{ \quad \}$

- **Disjoint sets** -- Two sets are said to be disjoint if they do not have a common element, i.e. no element in A is in B and no element in B is in A.



- $A \cap B = \phi$

- **Set difference** – the set difference of two sets A and B, is the set of all the elements which belongs to A but do not belong to B.
- $A - B = \{x \mid x \in A \text{ and } x \notin B\}$

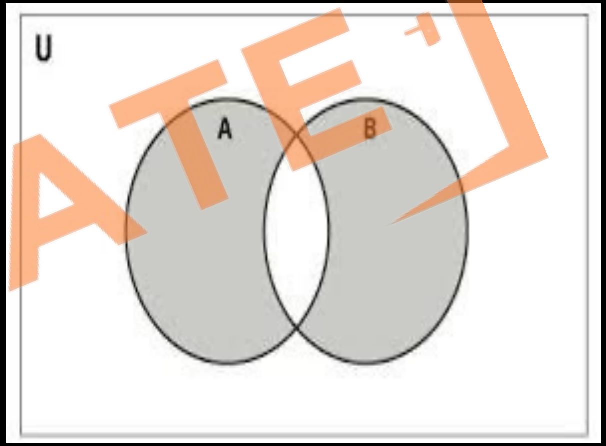


$A = \{1, 2, 3, 4\}$

$B = \{3, 4, 5, 6\}$

$A - B = \{ \qquad \}$

Symmetric difference – the symmetric difference of two sets A and B is the set of all the elements that are in A or in B but not in both, denoted as. $A \oplus B = (A \cup B) - (A \cap B)$



$A \oplus B = \{x \mid (x \in A \text{ and } x \notin B) \text{ or } (x \in B \text{ and } x \notin A)\}$

$A \oplus B = (A - B) \cup (B - A)$

$A = \{1, 2, 3, 4\}$

$B = \{3, 4, 5, 6\}$

$A \oplus B = \{ \qquad \qquad \}$

Q Consider the following statements?

a) Finite union of finite sets(disjoint) is ________(finite/infinite)

b) Finite union of Infinite sets(disjoint) is ________(finite/infinite)

c) Infinite union of finite sets(distinct) is ________(finite/infinite)

d) if after finite number of union result is infinite set, then at least of the input set(disjoint) is infinite (T / F)

e) if after finite number of union result is infinite set, then all of the input set is infinite (T / F)

f) Finite intersection of finite sets(disjoint) is _________(finite/infinite)

g) Finite intersection of Infinite sets(disjoint) is _________(finite/infinite)

h) If after finite number of intersection result is infinite set, then at least of the input set(disjoint) is infinite (T / F)

i) If after finite number of intersection result is infinite set, then all of the input set(disjoint) is infinite (T / F)

Q Let S be an infinite set S_1, S_2……….., S_n be Sets such that $S_1 \cup S_2 \cup \ldots \cup S_n = S$ Then,

(a) At least one of the set S_i is a finite set

(b) Not more than one of the set S_i can be finite

(c) At least one of the sets S_i is an infinite set

(d) Not more than one of the sets S_i can be infinite

Q which of the following is not true?

a) $A - B = A \cap B^c$

b) $A - (A - B) = A \cap B$

c) $A - (A \cap B) = A - B$

d) $A - (A - B) = B$

Q If $A \subset B$, then which of the following is not true?

(a) $A \cup B = B$

(b) $A \cap B = A$

(c) $B^C \subset A^C$

(d) $B - A = \phi$

Q Which of the following is true?

(i) $(A - B) - C = A - (C - B)$

(ii) $(A - B) - C = (A - C) - B$

(iii) $(A - B) - C = A - (B \cap C)$

(iv) $(A \cap B) - (B \cap C) = \{A - (A \cap C)\} - (A - B)$

a) i & iii **b)** ii & iv

c) i, ii, iv **d)** ii & iii

Q If P, Q, R are subsets of the universal set U, then

$(P \cap Q \cap R) \cup (P^C \cap Q \cap R) \cup Q^C \cup R^C$

(A) $Q^c \cup R^c$

(B) $P \cup Q^c \cup R^c$

(C) $P^c \cup Q^c \cup R^c$

(D) U

Q let p, q and r be sets let @ denotes the symmetric difference operator defined as

$P @ q = (p \cup q) - (p \cap q)$?

I) $p @ (q \cap r) = (p @ q) \cap (P @ r)$

II) $p \cap (q \cap r) = (p \cap q) @ (p @ r)$

a) I only **b)** II only

c) neither I nor II **d)** both I and II

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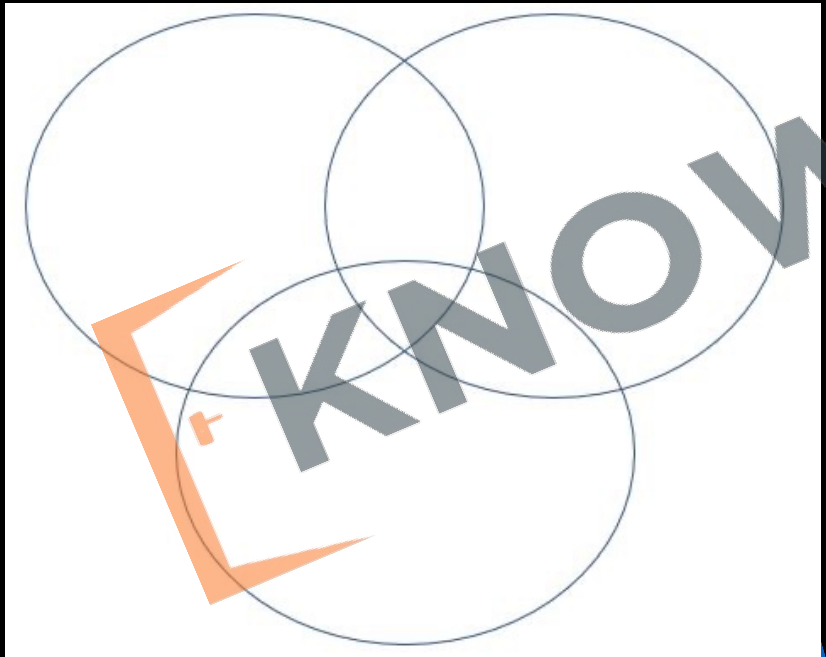
Q Let E, F and G be finite sets.

Let $X = (E \cap F) - (F \cap G)$ and $Y = (E - (E \cap G)) - (E - F)$.

Which one of the following is true?

(A) $X \subset Y$ **(B)** $X \supset Y$ **(C)** $X = Y$ **(D)** $X - Y \neq \phi$ and $Y - X \neq \phi$



Q what is the cardinality of the set of integers X defined below

X = {n| 1<=n<=123, n is not divisible by 2,3 or 5}?

a)90

b)33

c)37

d)44

Q Let A, B and C be non-empty sets and let

$X=(A-B)-C$

$Y=(A-C)-(B-C)$

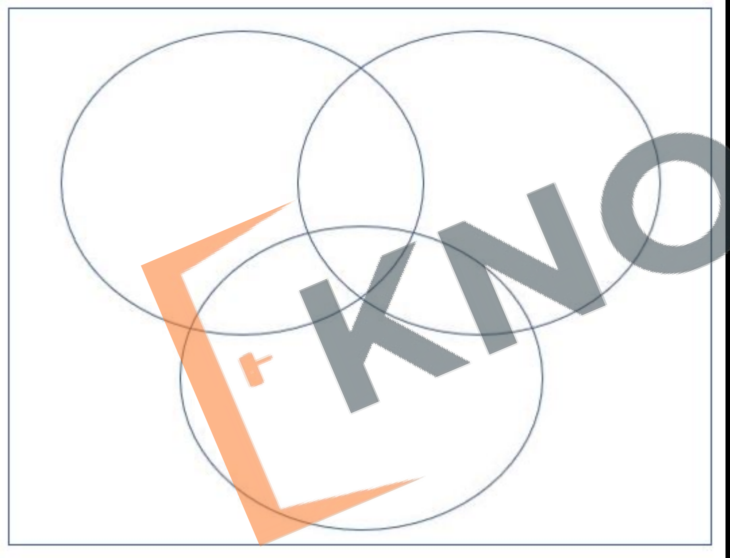
Which one of the following is TRUE?

a) $X=Y$ **b)** $X\subset Y$ **c)** $Y\subset X$ **d)** None of these

Q In a class of 200 students, 125 students have taken Programming Language course, 85 students have taken Data Structures course, 65 students have taken Computer Organization course; 50 students have taken both Programming Language and Data Structures, 35 students have taken both Data Structures and Computer Organization; 30 students have taken both Programming Language and Computer Organization, 15 students have taken all the three courses. How many students have not taken any of the three courses?

(A) 175 **(B)** 20 **(C)** 25 **(D)** 35

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |A \cap C| + |A \cap B \cap C|$$



Q Let A and B be sets and let A^c and B^c denote the complements of the sets A and B. the set $(a - b) \cup (b - a) \cup (a \cap b)$ is equal to.

(a) $A \cup B$ **(b)** $A^c \cup b^c$ **(c)** $A \cap B$ **(d)** $A^c \cap b^c$

Idempotent law

- $A \cup A = A$
- $A \cap A = A$

Associative law

- $(A \cup B) \cup C = A \cup (B \cup C)$
- $(A \cap B) \cap C = A \cap (B \cap C)$

Commutative law

- $A \cup B = B \cup A$
- $A \cap B = B \cap A$

Distributive law

- $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
- $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

De Morgan's law

- $(A \cup B)^C = A^C \cap B^C$
- $(A \cap B)^C = A^C \cup B^C$

Identity law

- $A \cup \phi = A$
- $A \cap \phi = \phi$
- $A \cup U = U$
- $A \cap U = A$

Complement law

- $A \cup A^C = U$
- $A \cap A^C = \phi$
- $U^C = \phi$
- $\phi^C = U$

Involution law

- $((A)^C)^C = A$

Chapter-2 (Relations)

Cartesian Product

- Cartesian Product of two sets A and B in the set of all ordered pairs, whose first member belongs to the first set and second member belongs to the second set, denoted by A × B. It is a kind of maximum relation possible, where every member of the first set belong to every member of the second set.
- $A \times B = \{(a, b) \mid a \in A \text{ and } b \in B\}$
- For E.g. if A = {a, b}, B = {1, 2, 3}
- A×B = { }

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1. In general, commutative law does not hold good $A \times B \neq B \times A$

2. If $|A| = m$ and $|B| = n$ then $|A \times B| =$

Relation

- **Relation**: - Let A and B are sets then every possible subset of 'A×B' is called a relation from A to B.

- If |A| =m and |B| = n then total no of element(pair) will be m*n, every element will have two choice weather to present or not present in the subset(relation), therefore the total number of relation possible is ________

- Largest relation possible will be ________

- Smallest possible relation will be ________

For E.g. if A = {a, b}, B = {1, 2}, A×B = {(a, 1), (a, 2), (b, 1), (b, 2)}

(a , 1)	(a , 2)	(b , 1)	(b , 2)		
0	0	0	0	0	
0	0	0	1	1	
0	0	1	0	2	
0	0	1	1	3	
0	1	0	0	4	
0	1	0	1	5	
0	1	1	0	6	
0	1	1	1	7	
1	0	0	0	8	
1	0	0	1	9	
1	0	1	0	10	
1	0	1	1	11	
1	1	0	0	12	
1	1	0	1	13	
1	1	1	0	14	
1	1	1	1	15	

For E.g. if A = {a, b}, B = {1, 2}, A×B = {(a, 1), (a, 2), (b, 1), (b, 2)}

(a , 1)	(a , 2)	(b , 1)	(b , 2)		
0	0	0	0	0	{ }
0	0	0	1	1	{(b, 2)}
0	0	1	0	2	{(b, 1)}
0	0	1	1	3	{(b, 1), (b, 2)}
0	1	0	0	4	{(a, 2)}
0	1	0	1	5	{(a, 2), (b, 2)}
0	1	1	0	6	{(a, 2), (b, 1)}
0	1	1	1	7	{(a, 2), (b, 1), (b, 2)}
1	0	0	0	8	{(a, 1)}
1	0	0	1	9	{(a, 1), (b, 2)}
1	0	1	0	10	{(a, 1), (b, 1)}
1	0	1	1	11	{(a, 1), (b, 1), (b, 2)}
1	1	0	0	12	{(a, 1), (a, 2)}
1	1	0	1	13	{(a, 1), (a, 2), (b, 2)}
1	1	1	0	14	{(a, 1), (a, 2), (b, 1)}
1	1	1	1	15	{(a, 1), (a, 2), (b, 1), (b, 2)}

- **Complement of a relation**: - Let R be a relation from A to B, then the complement of relation will be denoted by R', R^C or R.
- R' = {(a, b) | (a, b) ∊ A×B, (a, b) ∊! R}
- R' = (A×B) – R
- For E.g. if A×B = {(a, 1), (a, 2), (a, 3), (b, 1), (b, 2), (b, 3)}
- R= {(a, 1), (a, 3), (b, 2)}
- R' = { }
- R U R' =
- R ∩ R' =

- **Inverse of a relation**: - Let R be a relation from A to B, then the inverse of relation will be a relation from B to A, denoted by R^{-1}.

- $R^{-1} = \{(b, a) \mid (a, b) \in R\}$

- $A \times B = \{(a, 1), (a, 2), (a, 3), (b, 1), (b, 2), (b, 3)\}$

- $R = \{(a, 1), (a, 3), (b, 2)\}$

- $R^{-1} = \{ \quad \}$

- $|R| \quad |R^{-1}|$

- **Diagonal relation**: - A relation R on a set A is said to be diagonal relation if, R is a set of all ordered pair (x, x), for every ∀x ∈ A, sometimes it is also denoted by $\blacktriangle_A$

- R = {(x, x) | ∀ x ∈ A}

	1	2	3
1	11		
2		22	
3			33

Types of a Relation

- To further study types of relations, we consider a set A with n elements, then a cartesian product A×A will have n^2 elements(pairs). Therefore, total number of relation possible is $2^{n * n}$

- **Reflexive relation**: - A relation R on a set A is said to be reflexive,
- If $\forall x \in A$
  - $(x, x) \in R$

	1	2	3
1	11		
2		22	
3			33

Q consider a set A = {1,2,3}, find which of the following relations are reflexive and Irreflexive?

	Relation	Reflexive	Irreflexive
1	$A \times A$		
2	ϕ		
3	{(1,1), (2,2), (3,3)}		
4	{(1,2), (2,3), (1,3)}		
5	{(1,1), (1,2), (2,1), (2,2)}		
6	{(1,1), (2,2), (3,3), (1,3), (2,1)}		
7	{(1,3), (2,1), (2,3), (3,2)}		

1. Smallest reflexive relation is ________

2. Largest reflexive relation is ________

3. Total number of reflexive relations will be _________

4. If two relations R_1 and R_2 are reflexive then their union and intersection will also be reflexive (T / F).

5. Any super set of reflexive relation will also be reflexive(T / F).

6. If a relation is reflexive then its inverse R^{-1} will also be reflexive (T / F).

For E.g. if A = {a, b}, A×A = {(a, a), (a, b), (b, a), (b, b)}

(a , a)	(a , b)	(b , a)	(b , b)		
0	0	0	0	0	{ }
0	0	0	1	1	{(b, b)}
0	0	1	0	2	{(b, a)}
0	0	1	1	3	{(b, a), (b, b)}
0	1	0	0	4	{(a, b)}
0	1	0	1	5	{(a, b), (b, b)}
0	1	1	0	6	{(a, b), (b, a)}
0	1	1	1	7	{(a, b), (b, a), (b, b)}
1	0	0	0	8	{(a, a)}
1	0	0	1	9	{(a, a), (b, b)}
1	0	1	0	10	{(a, a), (b, a)}
1	0	1	1	11	{(a, a), (b, a), (b, b)}
1	1	0	0	12	{(a, a), (a, b)}
1	1	0	1	13	{(a, a), (a, b), (b, b)}
1	1	1	0	14	{(a, a), (a, b), (b, a)}
1	1	1	1	15	{(a, a), (a, b), (b, a), (b, b)}

For E.g. if A = {a, b}, A×A = {(a, a), (a, b), (b, a), (b, b)}

(a , a)	(a , b)	(b , a)	(b , b)		
0	0	0	0	0	
0	0	0	1	1	
0	0	1	0	2	
0	0	1	1	3	
0	1	0	0	4	
0	1	0	1	5	
0	1	1	0	6	
0	1	1	1	7	
1	0	0	0	8	
1	0	0	1	9	{(a, a), (b, b)}
1	0	1	0	10	
1	0	1	1	11	{(a, a), (b, a), (b, b)}
1	1	0	0	12	
1	1	0	1	13	{(a, a), (a, b), (b, b)}
1	1	1	0	14	
1	1	1	1	15	{(a, a), (a, b), (b, a), (b, b)}

- **Irreflexive relation**: - A relation R on a set A is said to be Irreflexive,
  1. If $\forall x \in A$
  2. $(x, x) \notin R$

Q consider a set A = {1,2,3}, find which of the following relations are reflexive and Irreflexive?

	Relation	Reflexive	Irreflexive
1	$A \times A$	Y	
2	ϕ	N	
3	{(1,1), (2,2), (3,3)}	Y	
4	{(1,2), (2,3), (1,3)}	N	
5	{(1,1), (1,2), (2,1), (2,2)}	N	
6	{(1,1), (2,2), (3,3), (1,3), (2,1)}	Y	
7	{(1,3), (2,1), (2,3), (3,2)}	N	

1. Smallest irreflexive relation is ________

2. Largest irreflexive relation is _______

3. Total number of irreflexive relation will be ________

4. If two relations R_1 and R_2 are Irreflexive then their union and intersection will also be Irreflexive (T / F).

5. If a relation R on a set A is reflexive, then R^C is Irreflexive, and vice versa (T / F).

6. Any sub set of irreflexive relation will also be irreflexive(T / F).

7. If a relation is irreflexive then its inverse R^{-1} will also be irreflexive (T / F).

For E.g. if A = {a, b}, A×A = {(a, a), (a, b), (b, a), (b, b)}

(a , a)	(a , b)	(b , a)	(b , b)		
0	0	0	0	0	{ }
0	0	0	1	1	{(b, b)}
0	0	1	0	2	{(b, a)}
0	0	1	1	3	{(b, a), (b, b)}
0	1	0	0	4	{(a, b)}
0	1	0	1	5	{(a, b), (b, b)}
0	1	1	0	6	{(a, b), (b, a)}
0	1	1	1	7	{(a, b), (b, a), (b, b)}
1	0	0	0	8	{(a, a)}
1	0	0	1	9	{(a, a), (b, b)}
1	0	1	0	10	{(a, a), (b, a)}
1	0	1	1	11	{(a, a), (b, a), (b, b)}
1	1	0	0	12	{(a, a), (a, b)}
1	1	0	1	13	{(a, a), (a, b), (b, b)}
1	1	1	0	14	{(a, a), (a, b), (b, a)}
1	1	1	1	15	{(a, a), (a, b), (b, a), (b, b)}

For E.g. if A = {a, b}, A×A = {(a, a), (a, b), (b, a), (b, b)}

(a , a)	(a , b)	(b , a)	(b , b)		
0	0	0	0	0	{ }
0	0	0	1	1	
0	0	1	0	2	{(b, a)}
0	0	1	1	3	
0	1	0	0	4	{(a, b)}
0	1	0	1	5	
0	1	1	0	6	{(a, b), (b, a)}
0	1	1	1	7	
1	0	0	0	8	
1	0	0	1	9	
1	0	1	0	10	
1	0	1	1	11	
1	1	0	0	12	
1	1	0	1	13	
1	1	1	0	14	
1	1	1	1	15	

- **Symmetric relation**: - A relation R on a set A is said to be Symmetric,

If $\forall$ a, b $\in$ A

(a, b) $\in$ R

………………………………

then (b, a) $\in$ R

………………………………

Q Consider a set A = {1,2,3}, find which of the following relations are Symmetric, Anti-Symmetric and Asymmetric?

	Relation	Symmetric	Anti-Symmetric	Asymmetric
1	$A \times A$			
2	ϕ			
3	{(1,1), (2,2), (3,3)}			
4	{(1,2), (2,3), (1,3)}			
5	{(1,1), (1,2), (2,1), (2,2)}			
6	{(1,1), (2,2), (3,3), (1,3), (2,1)}			
7	{(1,3), (2,1), (2,3), (3,2)}			

1. Smallest symmetric relation is ________

2. Largest symmetric relation is_______

3. Total number of symmetric relation will be _________

4. If a relation on a set A is symmetric then R __ R^{-1}

5. If two relations R_1 and R_2 are symmetric then their Union and Intersection will also be symmetric. (T / F)

6. If a relation is symmetric then its superset and subset will always be symmetric. (T / F)

7. If a relation is symmetric then its complement R^c will always be symmetric. (T / F)

For E.g. if A = {a, b}, A×A = {(a, a), (a, b), (b, a), (b, b)}

(a , a)	(a , b)	(b , a)	(b , b)		
0	0	0	0	0	{ }
0	0	0	1	1	{(b, b)}
0	0	1	0	2	{(b, a)}
0	0	1	1	3	{(b, a), (b, b)}
0	1	0	0	4	{(a, b)}
0	1	0	1	5	{(a, b), (b, b)}
0	1	1	0	6	{(a, b), (b, a)}
0	1	1	1	7	{(a, b), (b, a), (b, b)}
1	0	0	0	8	{(a, a)}
1	0	0	1	9	{(a, a), (b, b)}
1	0	1	0	10	{(a, a), (b, a)}
1	0	1	1	11	{(a, a), (b, a), (b, b)}
1	1	0	0	12	{(a, a), (a, b)}
1	1	0	1	13	{(a, a), (a, b), (b, b)}
1	1	1	0	14	{(a, a), (a, b), (b, a)}
1	1	1	1	15	{(a, a), (a, b), (b, a), (b, b)}

For E.g. if A = {a, b}, A×A = {(a, a), (a, b), (b, a), (b, b)}

(a , a)	(a , b)	(b , a)	(b , b)		
0	0	0	0	0	{ }
0	0	0	1	1	{(b, b)}
0	0	1	0	2	
0	0	1	1	3	
0	1	0	0	4	
0	1	0	1	5	
0	1	1	0	6	{(a, b), (b, a)}
0	1	1	1	7	{(a, b), (b, a), (b, b)}
1	0	0	0	8	{(a, a)}
1	0	0	1	9	{(a, a), (b, b)}
1	0	1	0	10	
1	0	1	1	11	
1	1	0	0	12	
1	1	0	1	13	
1	1	1	0	14	{(a, a), (a, b), (b, a)}
1	1	1	1	15	{(a, a), (a, b), (b, a), (b, b)}

- **Anti-Symmetric relation**: - A relation R on a set A with cartesian product A×A is said to be Anti-Symmetric,

If $\forall$ a, b $\in$ A

(a, b) $\in$ R

(b, a) $\in$ R

……………………………

a = b

……………………………

Conclusion: Symmetry is not allowed but diagonal pairs are allowed

	Relation	Symmetric	Anti-Symmetric	Asymmetric
1	A×A	Y		
2	ϕ	Y		
3	{(1,1), (2,2), (3,3)}	Y		
4	{(1,2), (2,3), (1,3)}	N		
5	{(1,1), (1,2), (2,1), (2,2)}	Y		
6	{(1,1), (2,2), (3,3), (1,3), (2,1)}	N		
7	{(1,3), (2,1), (2,3), (3,2)}	N		

1. Smallest Anti-symmetric relation is __________

2. Largest Anti-symmetric relation will contain _________elements

3. Total number of Anti-symmetric relation will be __________

4. A relation R on a set A is Anti-Symmetric if $(R \cap R^{-1}) \sqsubseteq \blacktriangle_A$ (T / F)

5. Sub set of a Anti-Symmetric will also be (T / F)

6. Super set of a Anti-Symmetric will also be (T / F)

7. If two relations R_1 and R_2 are Anti-symmetric then their __________need not to be Anti-symmetric but __________ will also be Anti-symmetric.

8. If a relation is Anti-symmetric then its complement R^c will always be Anti-symmetric. (T / F)

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For E.g. if A = {a, b}, A×A = {(a, a), (a, b), (b, a), (b, b)}

(a , a)	(a , b)	(b , a)	(b , b)		
0	0	0	0	0	{ }
0	0	0	1	1	{(b, b)}
0	0	1	0	2	{(b, a)}
0	0	1	1	3	{(b, a), (b, b)}
0	1	0	0	4	{(a, b)}
0	1	0	1	5	{(a, b), (b, b)}
0	1	1	0	6	{(a, b), (b, a)}
0	1	1	1	7	{(a, b), (b, a), (b, b)}
1	0	0	0	8	{(a, a)}
1	0	0	1	9	{(a, a), (b, b)}
1	0	1	0	10	{(a, a), (b, a)}
1	0	1	1	11	{(a, a), (b, a), (b, b)}
1	1	0	0	12	{(a, a), (a, b)}
1	1	0	1	13	{(a, a), (a, b), (b, b)}
1	1	1	0	14	{(a, a), (a, b), (b, a)}
1	1	1	1	15	{(a, a), (a, b), (b, a), (b, b)}

For E.g. if A = {a, b}, A×A = {(a, a), (a, b), (b, a), (b, b)}

(a , a)	(a , b)	(b , a)	(b , b)		
0	0	0	0	0	{ }
0	0	0	1	1	{(b, b)}
0	0	1	0	2	{(b, a)}
0	0	1	1	3	{(b, a), (b, b)}
0	1	0	0	4	{(a, b)}
0	1	0	1	5	{(a, b), (b, b)}
0	1	1	0	6	
0	1	1	1	7	
1	0	0	0	8	{(a, a)}
1	0	0	1	9	{(a, a), (b, b)}
1	0	1	0	10	{(a, a), (b, a)}
1	0	1	1	11	{(a, a), (b, a), (b, b)}
1	1	0	0	12	{(a, a), (a, b)}
1	1	0	1	13	{(a, a), (a, b), (b, b)}
1	1	1	0	14	
1	1	1	1	15	

Q Consider a set A = {a, b, c} and R_1, R_2, R_3 and R_4 are relations on A which of the following is not true?

		Symmetric	Anti-Symmetric	True
1	$R_1 = \{(a, a), (c, c)\}$	Y	Y	
2	$R_2 = \{(a, b), (b, a), (a, c)\}$	N	N	
3	$R_3 = \{(a, b), (b, c), (a, c)\}$	N	Y	
4	$R_4 = \{(a, b), (b, a), (c, c)\}$	Y	N	

Q Consider the binary relation $R = \{(x, y), (x, z), (z, x), (z, y)\}$ on the set $\{x, y, z\}$. Which one of the following is TRUE?

(A) R is symmetric but NOT antisymmetric

(B) R is NOT symmetric but antisymmetric

(C) R is both symmetric and antisymmetric

(D) R is neither symmetric nor antisymmetric

- **Asymmetric relation**: - A relation R on a set A is said to be Asymmetric,

If $\forall$ a, b $\in$ A

(a, b) $\in$ R

…………………………….

(b, a) $\notin$ R

…………………………….

Conclusion: Symmetry is not allowed; even diagonal pairs are not allowed

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	Relation	Symmetric	Anti-Symmetric	Asymmetric
1	A×A	Y	N	
2	ϕ	Y	Y	
3	{(1,1), (2,2), (3,3)}	Y	Y	
4	{(1,2), (2,3), (1,3)}	N	Y	
5	{(1,1), (1,2), (2,1), (2,2)}	Y	N	
6	{(1,1), (2,2), (3,3), (1,3), (2,1)}	N	Y	
7	{(1,3), (2,1), (2,3), (3,2)}	N	N	

1. Smallest Asymmetric relation is _______

2. Largest Asymmetric relation will contain __________elements

3. Total number of Asymmetric relation will be _________

4. Every asymmetric relation is also anti-symmetric (T / F)

5. Sub set of a Asymmetric will also be Asymmetric (T / F)

6. Super set of a Asymmetric will also be Asymmetric(T / F)

7. If two relations R_1 and R_2 are Asymmetric then their Union will also be Asymmetric(T / F).

8. If two relations R_1 and R_2 are Asymmetric then their Intersection will also be Asymmetric(T / F).

9. If a relation is Asymmetric then its complement R^c will always be Asymmetric. (T / F)

For E.g. if A = {a, b}, A×A = {(a, a), (a, b), (b, a), (b, b)}

(a , a)	(a , b)	(b , a)	(b , b)		
0	0	0	0	0	{ }
0	0	0	1	1	{(b, b)}
0	0	1	0	2	{(b, a)}
0	0	1	1	3	{(b, a), (b, b)}
0	1	0	0	4	{(a, b)}
0	1	0	1	5	{(a, b), (b, b)}
0	1	1	0	6	{(a, b), (b, a)}
0	1	1	1	7	{(a, b), (b, a), (b, b)}
1	0	0	0	8	{(a, a)}
1	0	0	1	9	{(a, a), (b, b)}
1	0	1	0	10	{(a, a), (b, a)}
1	0	1	1	11	{(a, a), (b, a), (b, b)}
1	1	0	0	12	{(a, a), (a, b)}
1	1	0	1	13	{(a, a), (a, b), (b, b)}
1	1	1	0	14	{(a, a), (a, b), (b, a)}
1	1	1	1	15	{(a, a), (a, b), (b, a), (b, b)}

For E.g. if A = {a, b}, A×A = {(a, a), (a, b), (b, a), (b, b)}

(a , a)	(a , b)	(b , a)	(b , b)		
0	0	0	0	0	{ }
0	0	0	1	1	
0	0	1	0	2	{(b, a)}
0	0	1	1	3	
0	1	0	0	4	{(a, b)}
0	1	0	1	5	
0	1	1	0	6	
0	1	1	1	7	
1	0	0	0	8	
1	0	0	1	9	
1	0	1	0	10	
1	0	1	1	11	
1	1	0	0	12	
1	1	0	1	13	
1	1	1	0	14	
1	1	1	1	15	

- **Transitive relation**: - A relation R on a set A is said to be Transitive,

If $\forall$ a, b, c $\in$ A

(a, b) $\in$ R

(b, c) $\in$ R

……………………………

(a, c) $\in$ R

……………………………

	Relation	Transitive
1	A×A	
2	ϕ	
3	{(1,1), (2,2), (3,3)}	
4	{(1,2), (2,3), (1,3)}	
5	{(1,1), (1,2), (2,1), (2,2)}	
6	{(1,1), (2,2), (3,3), (1,3), (2,1)}	
7	{(1,3), (2,1), (2,3), (3,2)}	
8	{(1,2)}	
9	{(1,3), (2,3)}	
10	{(1,2), (1,3)}	
11	{(2,3), (1,2)}	

1. Smallest Transitive relation is _______

2. Largest Transitive relation will contain ___________ elements

3. If two relations R_1 and R_2 are Transitive then their ________ need not to be transitive but ____________ will also be transitive.

For E.g. if A = {a, b}, A×A = {(a, a), (a, b), (b, a), (b, b)}

(a , a)	(a , b)	(b , a)	(b , b)		
0	0	0	0	0	{ }
0	0	0	1	1	{(b, b)}
0	0	1	0	2	{(b, a)}
0	0	1	1	3	{(b, a), (b, b)}
0	1	0	0	4	{(a, b)}
0	1	0	1	5	{(a, b), (b, b)}
0	1	1	0	6	{(a, b), (b, a)}
0	1	1	1	7	{(a, b), (b, a), (b, b)}
1	0	0	0	8	{(a, a)}
1	0	0	1	9	{(a, a), (b, b)}
1	0	1	0	10	{(a, a), (b, a)}
1	0	1	1	11	{(a, a), (b, a), (b, b)}
1	1	0	0	12	{(a, a), (a, b)}
1	1	0	1	13	{(a, a), (a, b), (b, b)}
1	1	1	0	14	{(a, a), (a, b), (b, a)}
1	1	1	1	15	{(a, a), (a, b), (b, a), (b, b)}

For E.g. if A = {a, b}, A×A = {(a, a), (a, b), (b, a), (b, b)}

(a , a)	(a , b)	(b , a)	(b , b)		
0	0	0	0	0	{ }
0	0	0	1	1	{(b, b)}
0	0	1	0	2	{(b, a)}
0	0	1	1	3	{(b, a), (b, b)}
0	1	0	0	4	{(a, b)}
0	1	0	1	5	{(a, b), (b, b)}
0	1	1	0	6	
0	1	1	1	7	
1	0	0	0	8	{(a, a)}
1	0	0	1	9	{(a, a), (b, b)}
1	0	1	0	10	{(a, a), (b, a)}
1	0	1	1	11	{(a, a), (b, a), (b, b)}
1	1	0	0	12	{(a, a), (a, b)}
1	1	0	1	13	{(a, a), (a, b), (b, b)}
1	1	1	0	14	
1	1	1	1	15	{(a, a), (a, b), (b, a), (b, b)}

|A| = n	No of transitive relation
0	1
1	2
2	13
3	171
4	3994

Warshall's Algorithm: -

Q Consider a set A = {1,2,3} and a relation **R** = {(1,1), (1,3), (2,2), (3,1), (3,2)}?

	1	2	3
Column			
Row			

	1	2	3
1			
2			
3			

Q Consider a set A = {1,2,3,4} and a relation

R = {(1, 1), (2, 1), (2, 2), (2, 3), (2, 4), (3, 1), (3, 2), (3, 3), (3, 4)}?

	1	2	3	4
Column				
Row				

	1	2	3	4
1				
2				
3				
4				

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Q The binary relation R= {(1,1), (2,1), (2,2), (2,3), (2,4), (3,1), (3,2), (3,3), (3,4)} on the set A = {1,2,3,4} is

(a) reflexive, symmetric and transitive

(b) neither reflexive, nor irreflexive but transitive

(c) irreflexive, symmetric and transitive

(d) irreflexive and antisymmetric

- **<u>Equivalence Relation</u>**: - A relation R on a set A with cartesian product A×A is said to be Equivalence, if it is

1. **Reflexive**
2. **Symmetric**
3. **Transitive**

- If two relations R_1 and R_2 are Equivalence then their union need not to be equivalence but intersection will also be Equivalence.

R_1 : (a, b) iff (a + b) is even over the set of integers

R_2 : (a, b) iff (a + b) is odd over the set of integers

R_3 : (a, b) iff a x b > 0 over the set of non-zero rational numbers

R_4 : (a, b) iff $|a - b| \leq 2$ over the set of natural numbers

Q Let S be a set of n elements. The number of ordered pairs in the largest and the smallest equivalence relations on S are

(A) n and n

(B) n^2 and n

(C) n^2 and 0

(D) n and 1

- **Equivalence Class:** - of an element is denoted by [x].
  $[x] = \{y \mid y \in A \text{ and } (x, y) \in R\}$ for all $x \in A$

- We can have $[x] = [y]$, even if $x \,!\!= y$

Q Consider A = {1, 2, 3, 4, 5} an equivalence relation R on A, R = {(1,1),(2,2),(3,3),(4,4),(5,5),(1,4),(4,1),(2,5),(5,2)} find the partition of a set A, defined by R.

[1] =

[2] =

[3] =

[4] =

[5] =

Partitions of a Set: - let A be a set, with n elements. Based on our understanding of equivalent classes, a subdivision of A into non-empty and non-overlapping subset is called a partition of A.

$A_1 \cup A_2 \cup A_3 \cup \dots\dots\dots\dots \cup A_n = A$

$A_1 \cap A_2 \cap A_3 \cap \dots\dots\dots\dots \cap A_n = \phi$

Q Consider A = {1, 2, 3, 4, 5} an equivalence relation R on A, R = {(1,1),(2,2),(3,3),(4,4),(5,5),(1,4),(4,1),(2,5),(5,2)} find the partition of a set A, defined by R.

[1] = {1, 4}

[2] = {2, 5}

[3] = {3}

[4] = {1,4}

[5] = {2, 5}

so we have partitions =

Q Let A = {1,2,3,4,5} is a set having partitions as {1, 4}, {2, 3, 5} , find the equivalence relation from which these partitions are created?

- **Partial Order Relation**: - A relation R on a set A with cartesian product $A \times A$ is said to be partial order, if it is

1. **Reflexive**
2. **Anti - Symmetric**
3. **Transitive**

- **Partial ordering set (Poset):** - a set A with partial ordering relation R defined on A is called a POSET and is denoted by [A, R]

- For e.g. [A, /], [A, <=], [P(S), $\sqsubseteq$]

- **Total order relation:** - A Poset [A, R] is called a total order set, if every pair of elements are comparable i.e. either (a, b) or (b, a) $\in$ R, for $\forall$ a, b $\in$ A

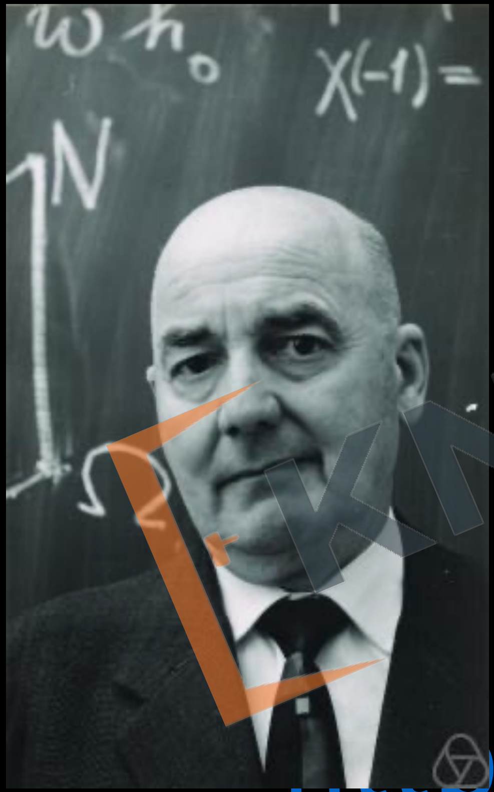
- For e.g. A = {1, 2, 3, 6}, then Poset [A,/] is not a total order relation but A = {1,2,4,8} will be

Chapter-3 (POSET & Lattices)

http://www.knowledgegate.in/gate

Conversion of POSET into a Hasse Diagram

- If we want to study Partial order relation further then it will be better to convert it into more convenient notation so that it can be studied easily. This graphical representation is called Hasse Diagram.
- The diagrams are named after Helmut Hasse (1898–1979).



Steps to convert partial order relation into hasse diagram

1- Draw a vertex for each element in the Set

2- If (a, b) ϵ R then draw an edge from a to b

3- Remove all Reflexive and Transitive edges

4- Remove the direction of edges and arrange them in the increasing order of heights.

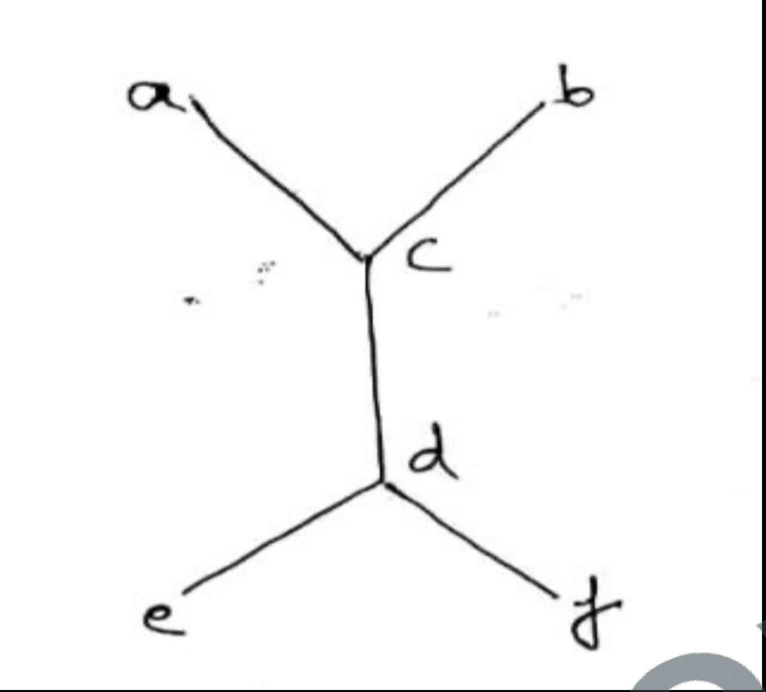
Q Consider a Partial order relation and convert it into hasse diagram?

R = {(1,1), (1,2), (1,4), (1,8), (2,2), (2,4), (2,8), (4,4), (4,8), (8,8)}

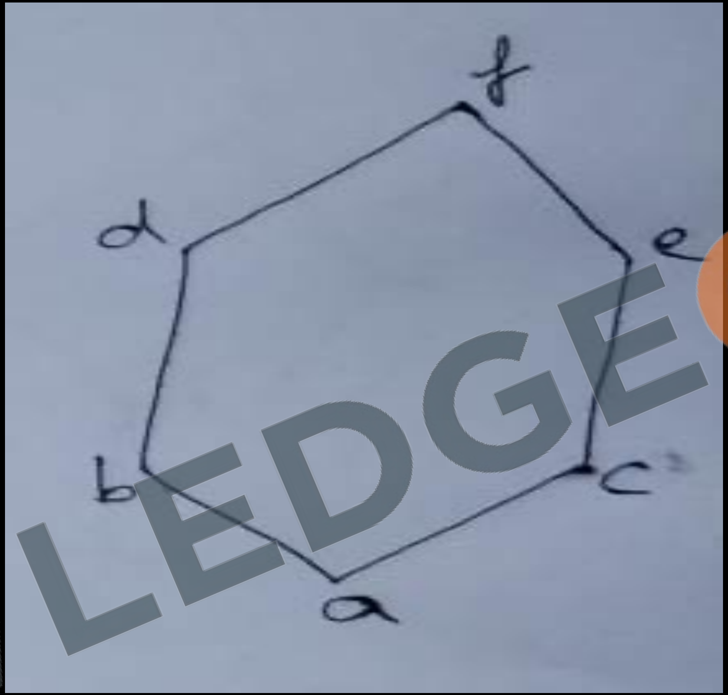
Q Consider a Partial order relation and convert it into hasse diagram?

R = {(1,1), (1,2), (1,3), (1,6), (2,2), (2,6), (3,3), (3,6), (6,6)}

Q Study the following hasse diagram and find which of the following are valid?



(1)



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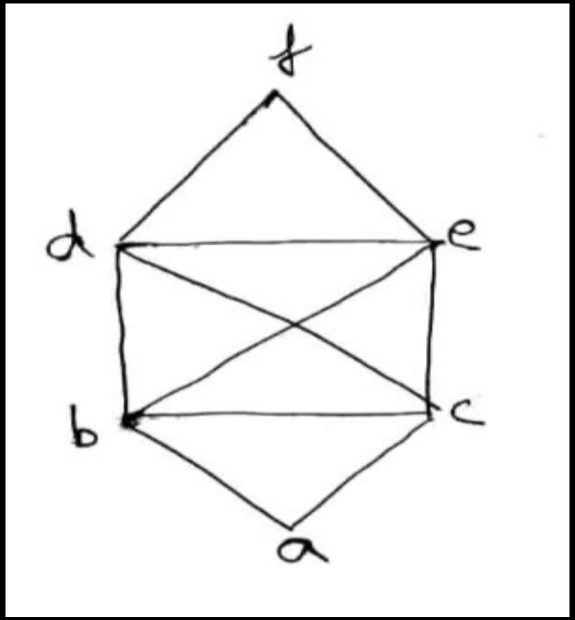

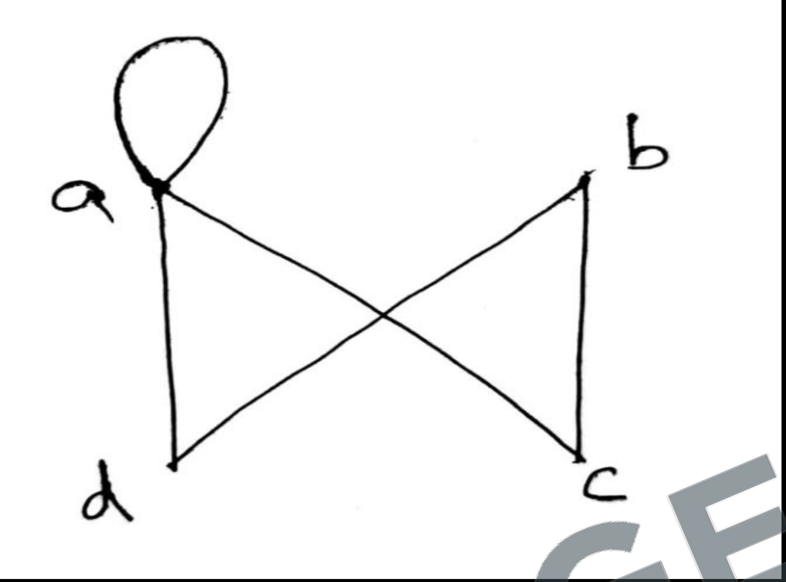

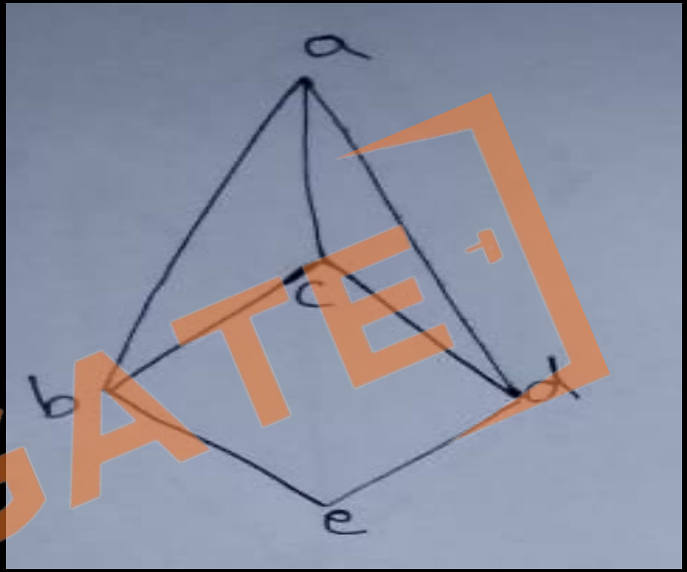

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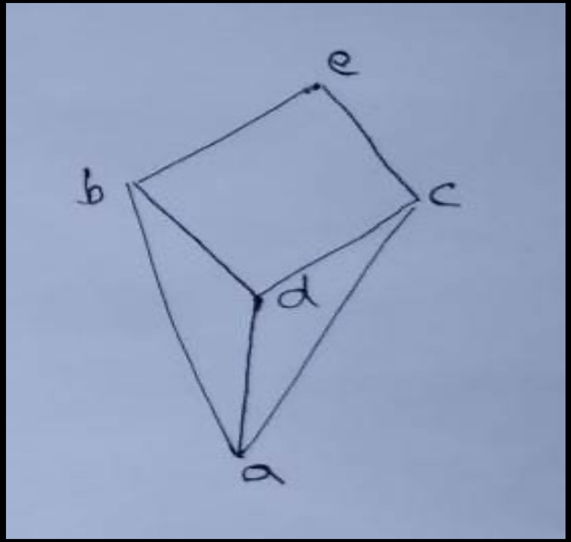

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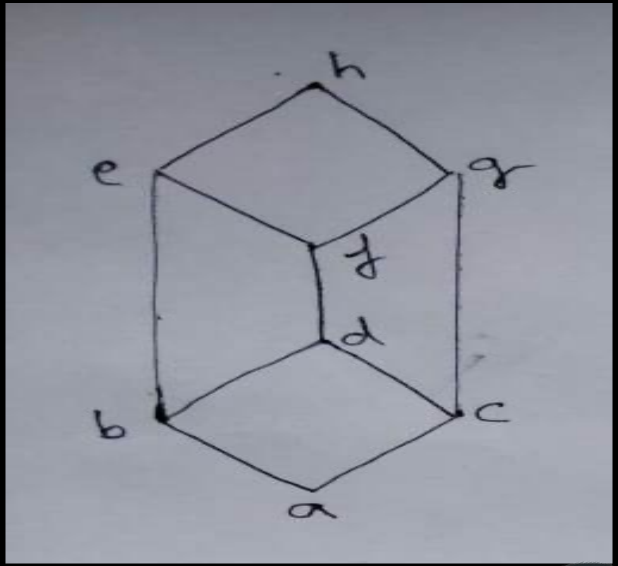

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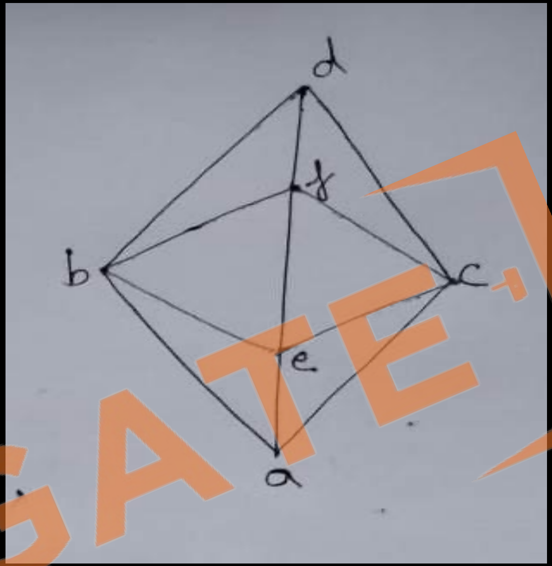

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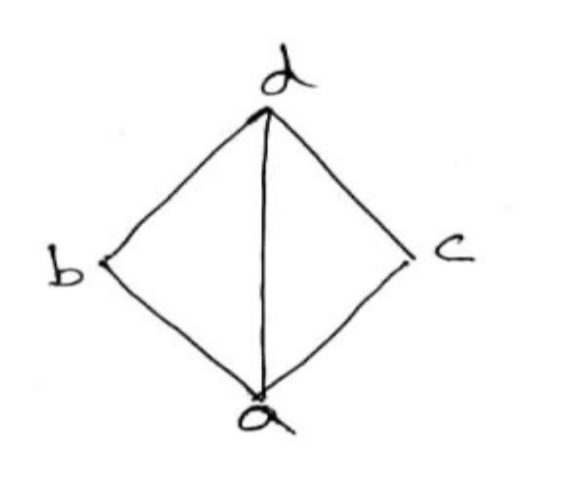

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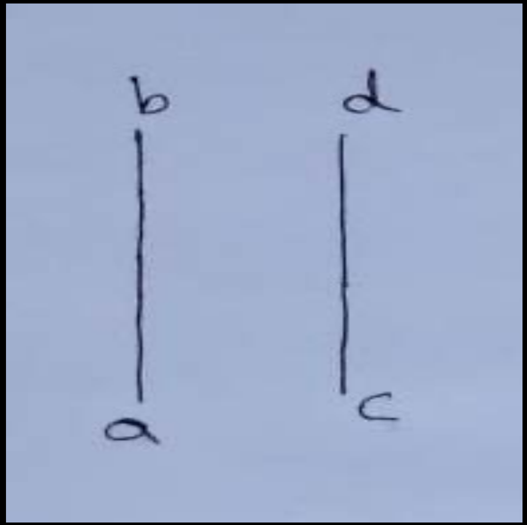

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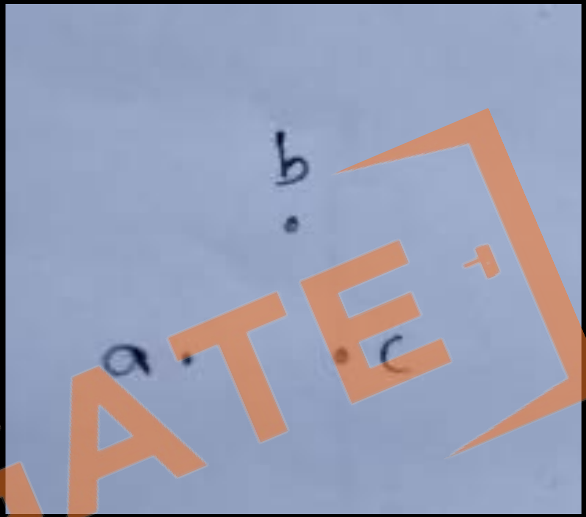

(10)

Conclusion

- We can not have a horizontal edge in a hasse diagram
- We can not have a reflexive and transitive edge in Hasse Diagram

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Q Let X = {2,3,6,12,24}, Let ≤ be the partial order defined by X ≤ Y if x divides y. Number of edges as in the Hasse diagram of (X, ≤) is.

(a) 3 **(b)** 4 **(c)** 9 **(d)** None of the above

Least Upper Bound / LUB / Join / Supremum / ∨

Least value in the upper bound

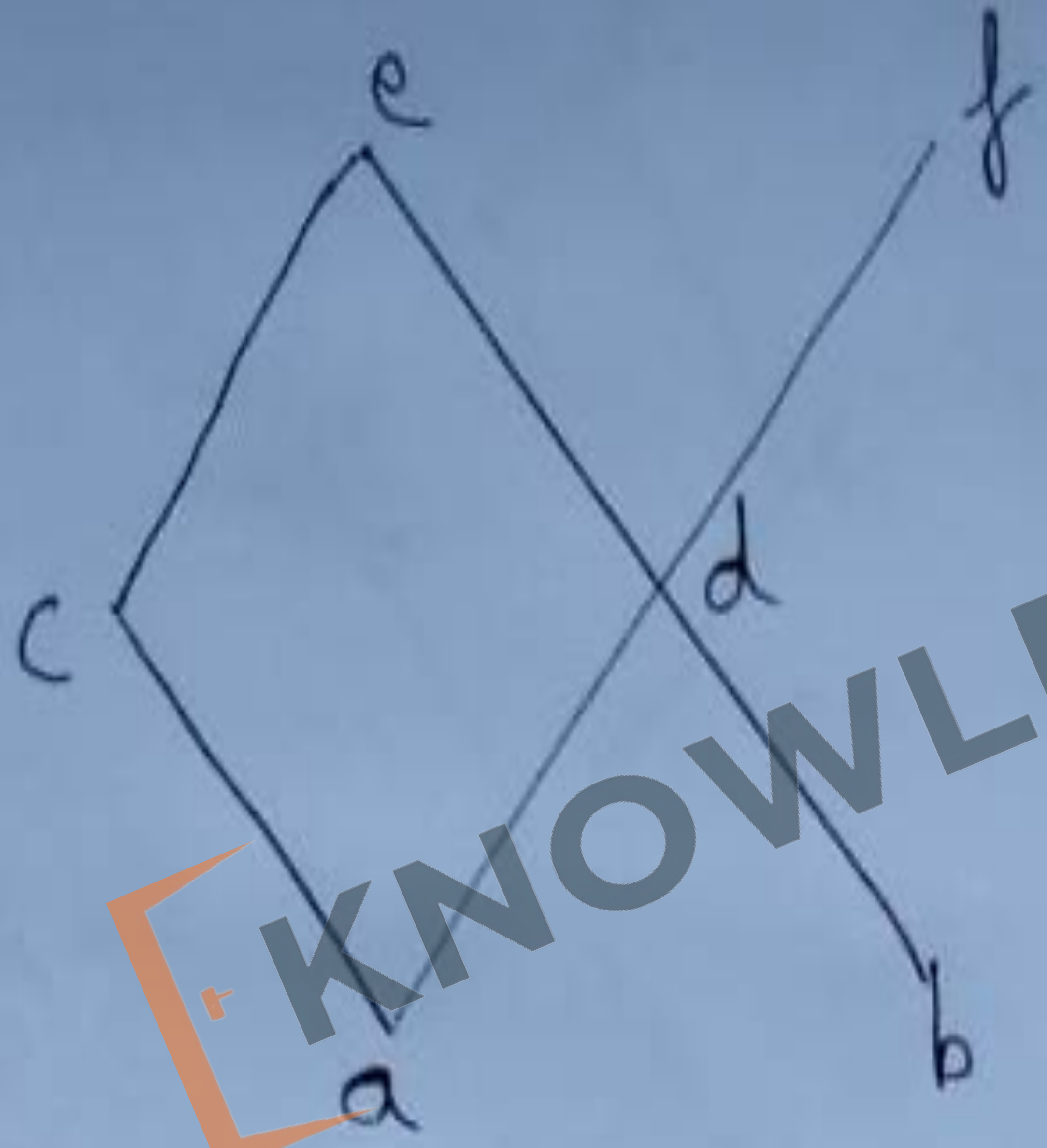
Greatest Lower Bound / GLB / Meet / Infimum / ∧

Greatest value in the lower bound

Join Semi Lattice :- A hasse diagram/Partial order relation is called Join Semi Lattice if for every elements their exists a Join.

Meet Semi Lattice :- A hasse diagram/Partial order relation is called Meet Semi Lattice if for every elements their exists a Meet.

Lattice :- A hasse diagram/Partial order relation is called Lattice if their exist a Join and Meet for every pair of element. Or A hasse diagram/Partial order relation is called Lattice if it is both Join Semi Lattice and Meet Semi Lattice.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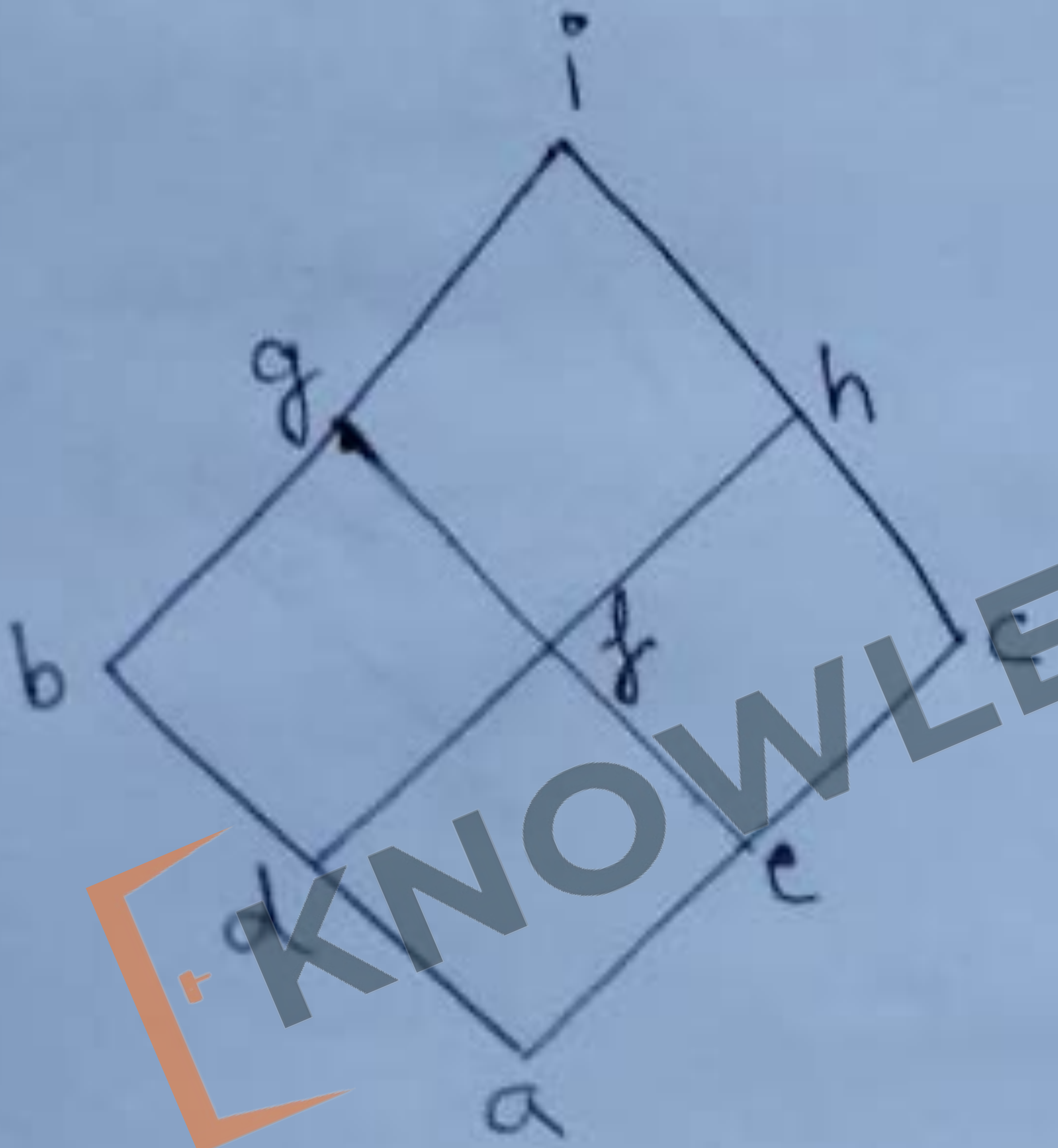
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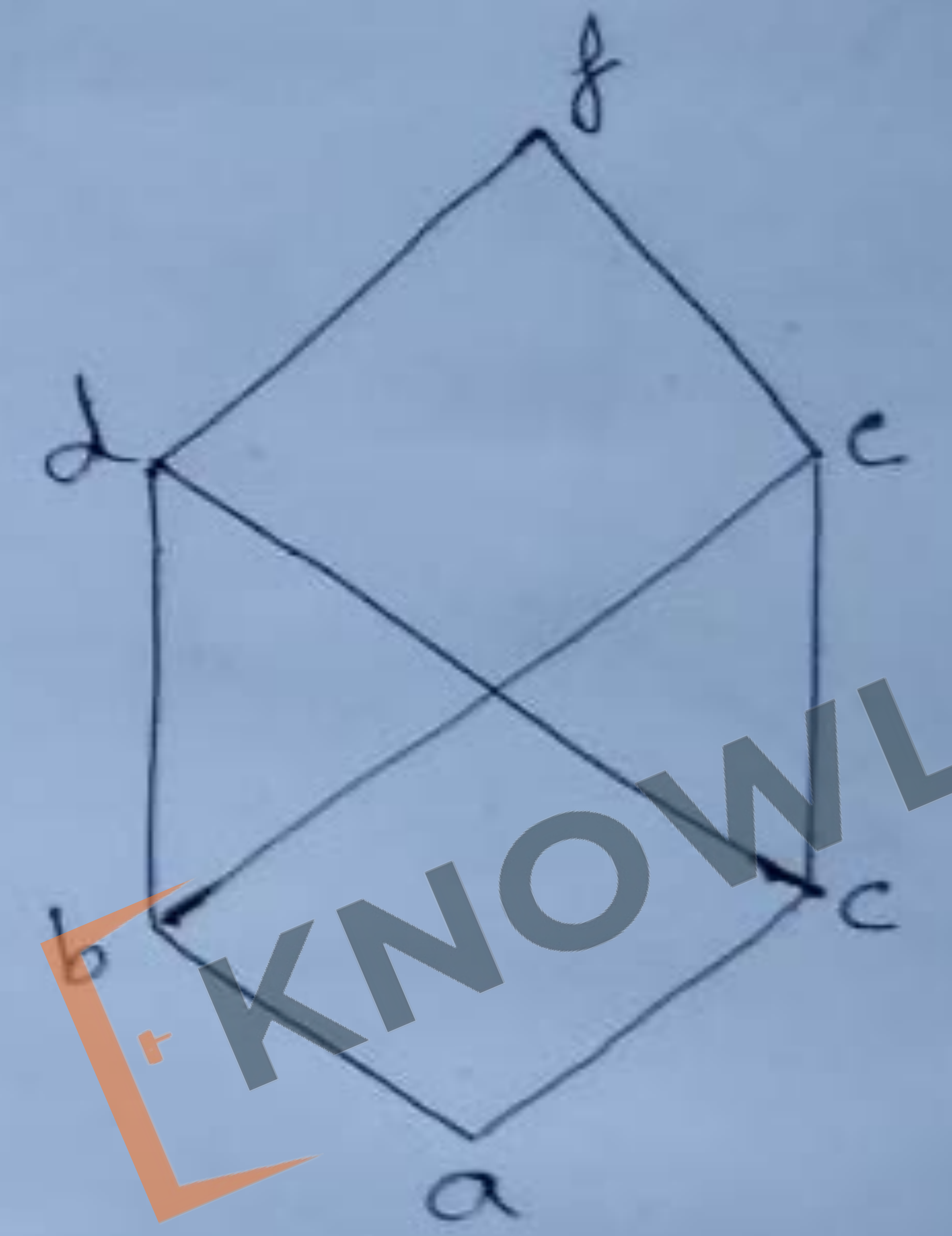
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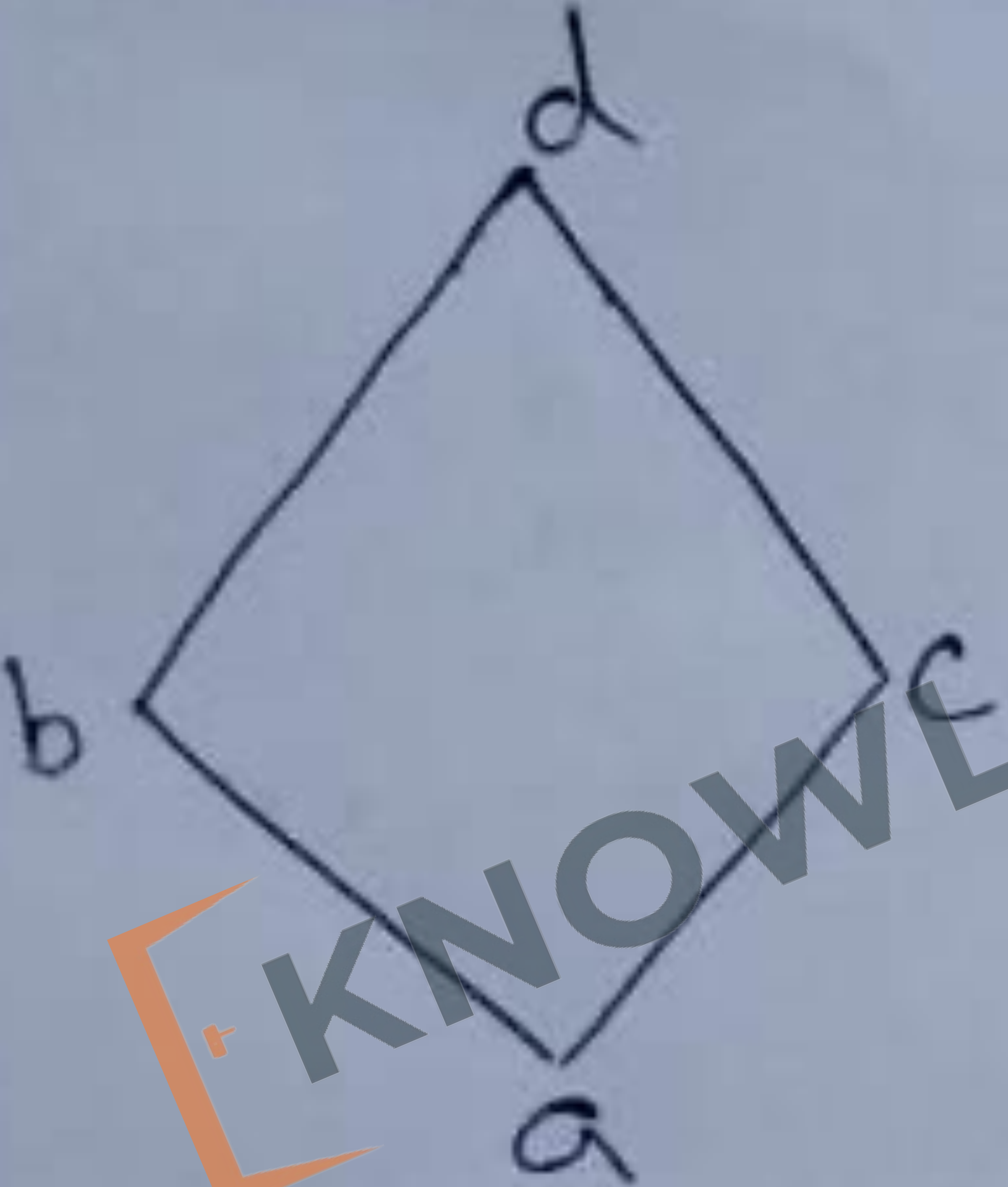
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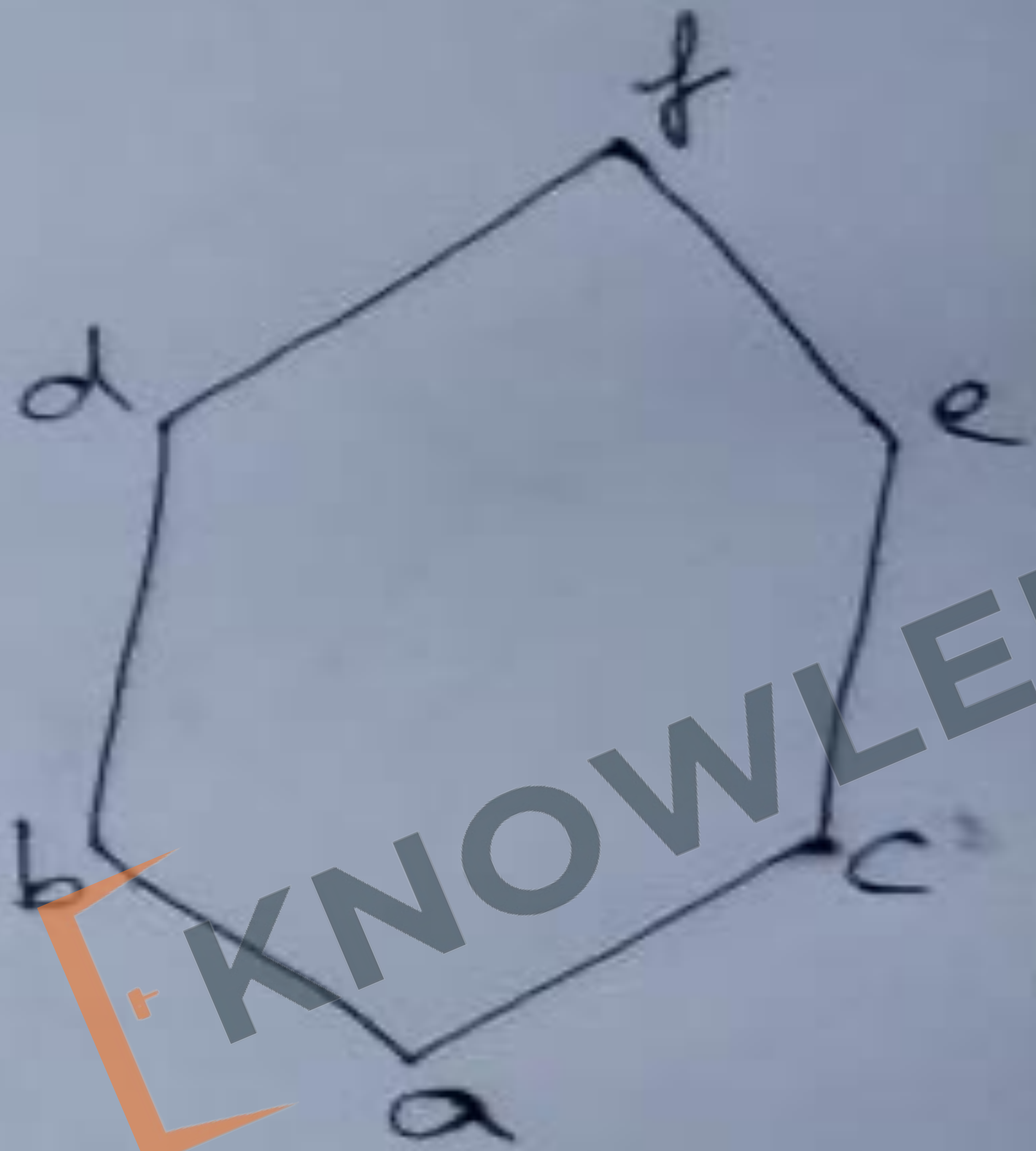
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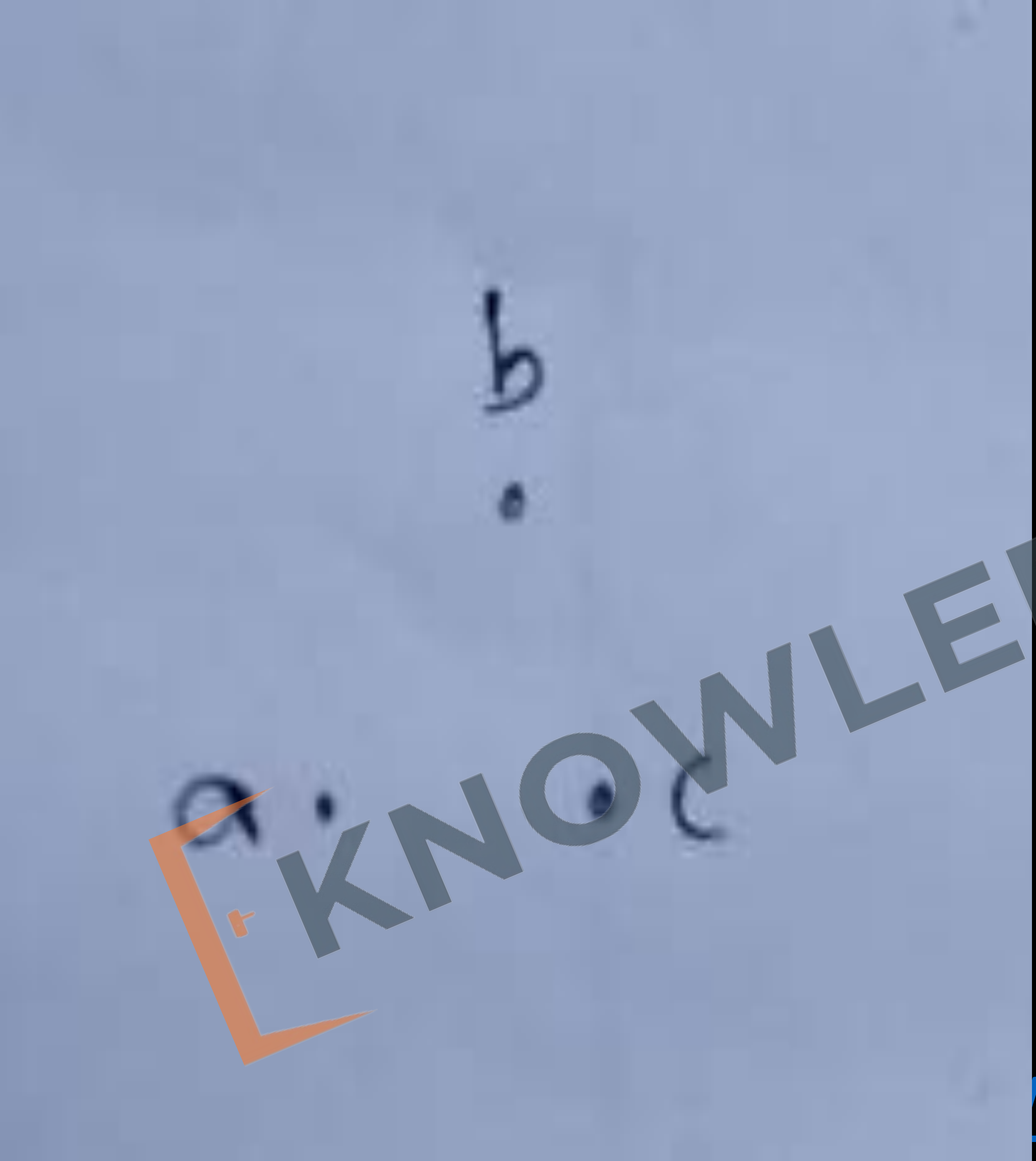
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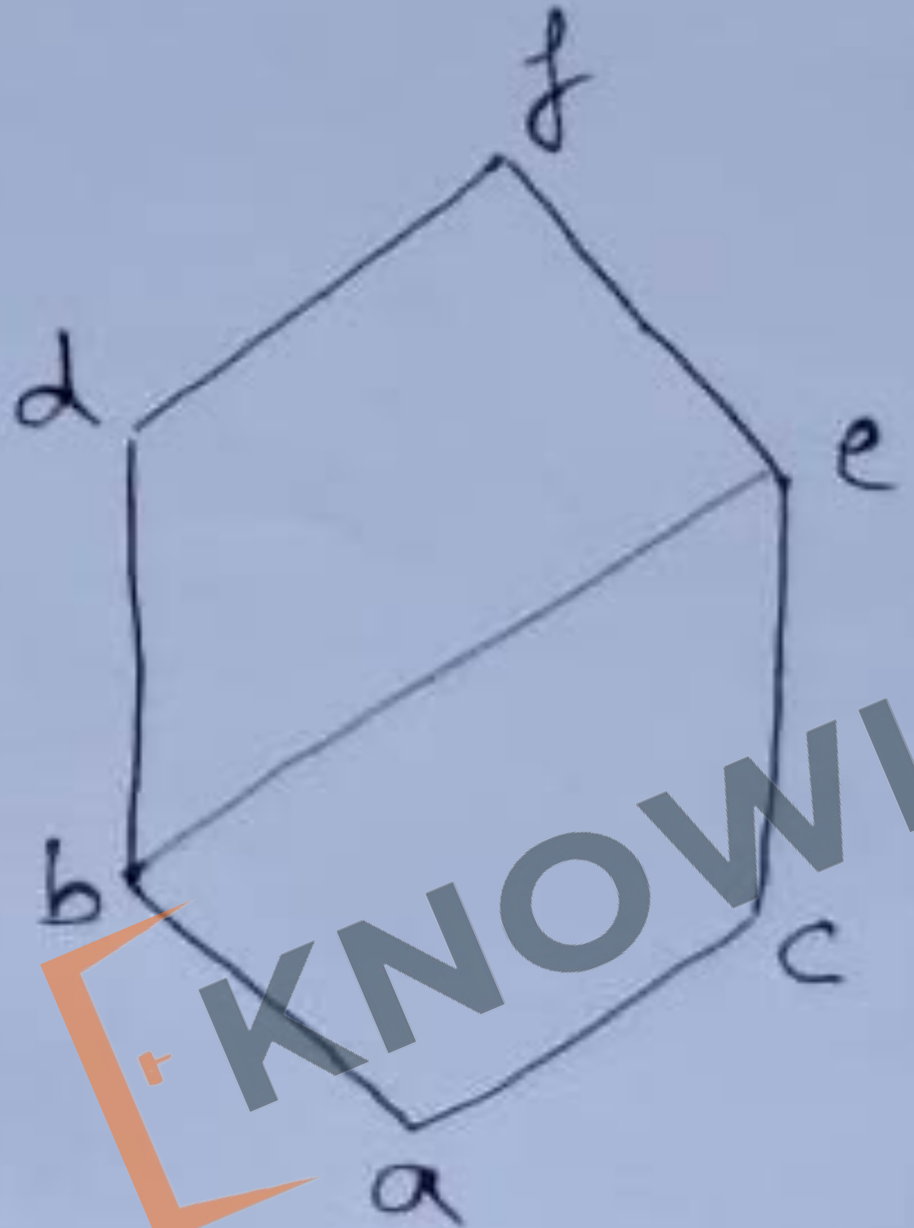
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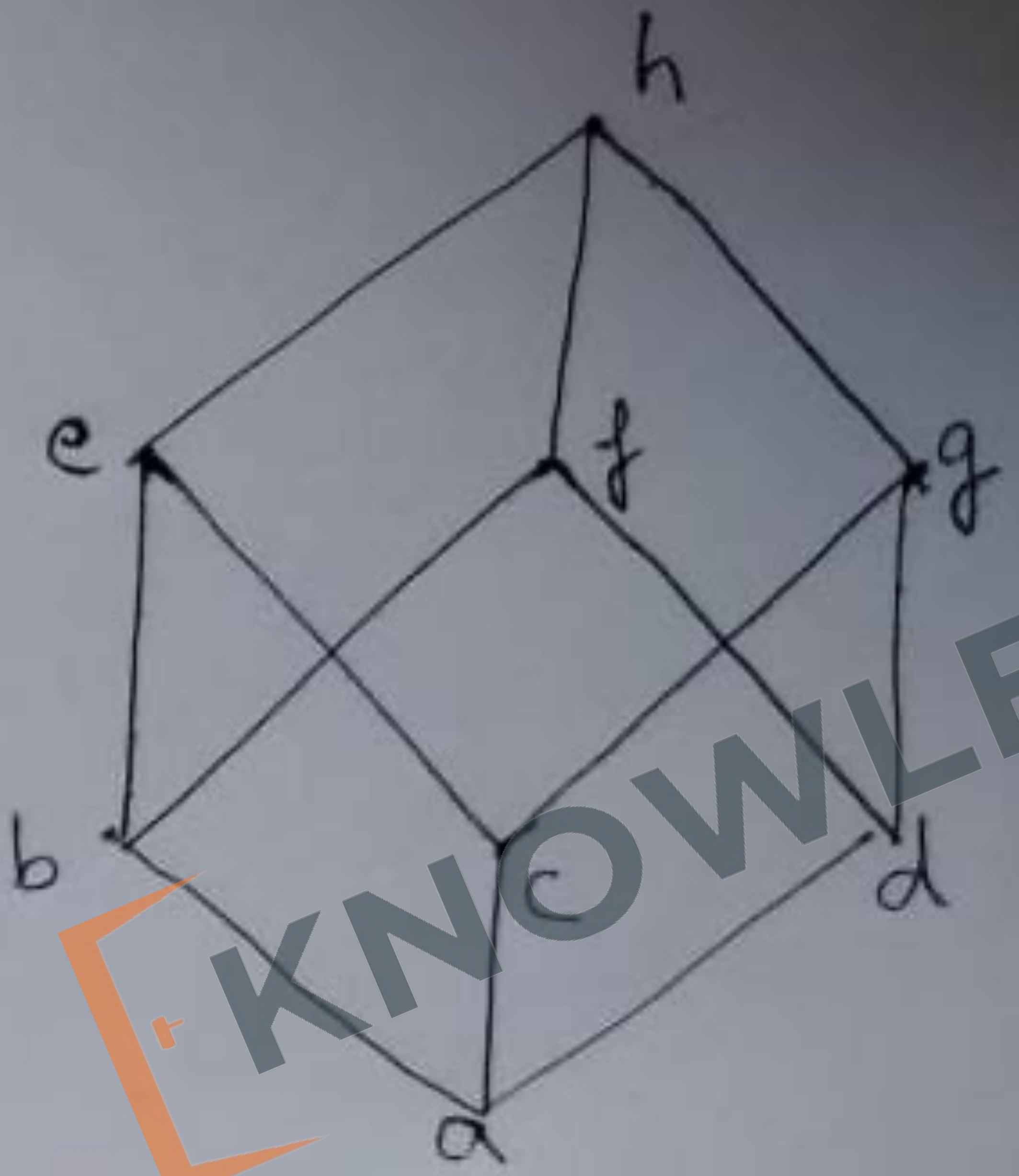
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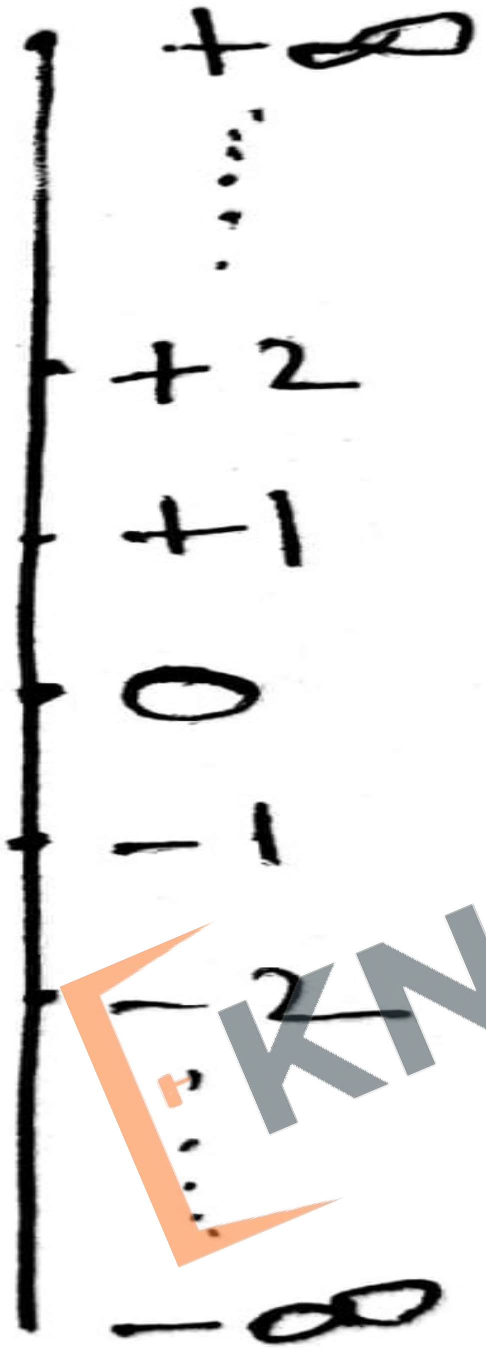
Boolean algebra

- **Unbounded Lattice :-** If a lattice has infinite of elements then it is called Unbounded Lattice.



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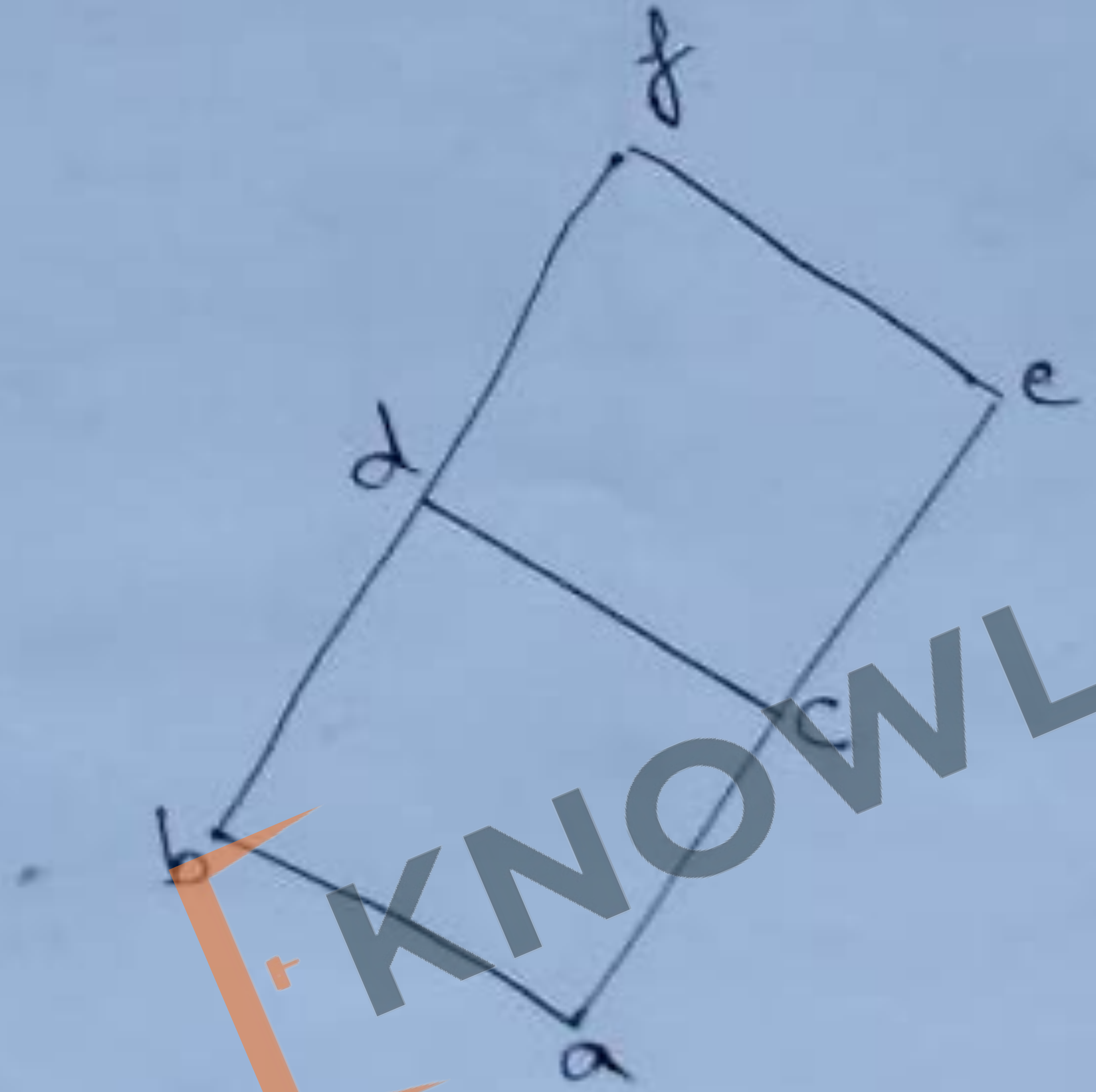
- **Bounded Lattice :-** If a lattice has finite number of elements then it is called Bounded lattice, there will be upper and lower bound in lattice.

- **Complement of an element in a Lattice :-** If two elements a and a^c, are complement of each other, then the following equations must always holds good.
  $a \vee a^c$ = Upper bound of lattice
  $a \wedge a^c$ = Lower bound of lattice

- **<u>Distributive Lattice</u> :-** A lattice is said to be distributed lattice. if for every element their exist at most one complement(zero or one).

- **<u>Complement Lattice</u> :-** A Lattice is said to be Complement lattice. if for every element their exist at least one complement(one or more).

- **<u>Boolean Algebra</u> :-** A Lattice is said to be Boolean Algebra, if for every element their exist exactly one complement. Or if a lattice is both complemented and distributed then it is called Boolean Algebra.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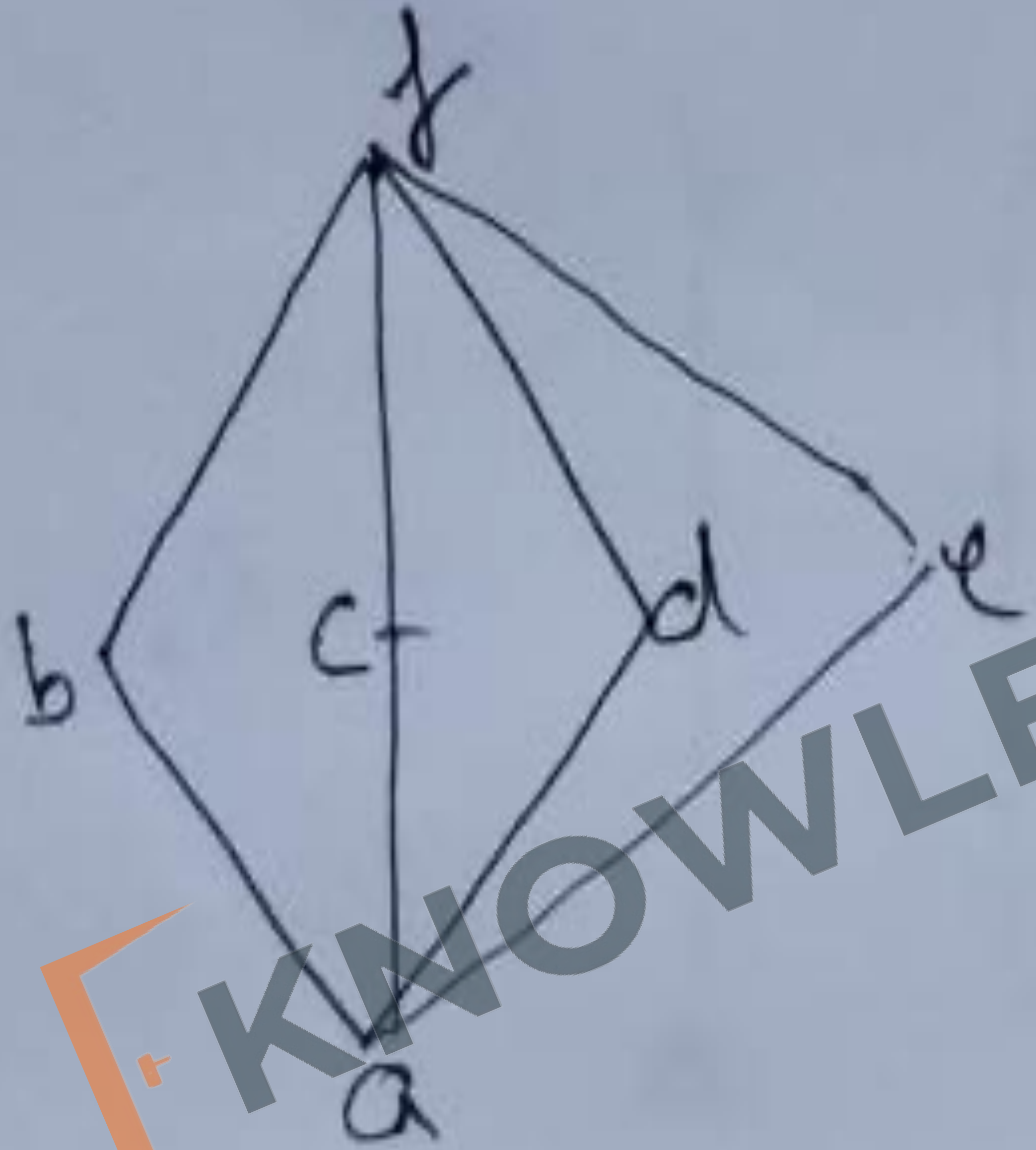
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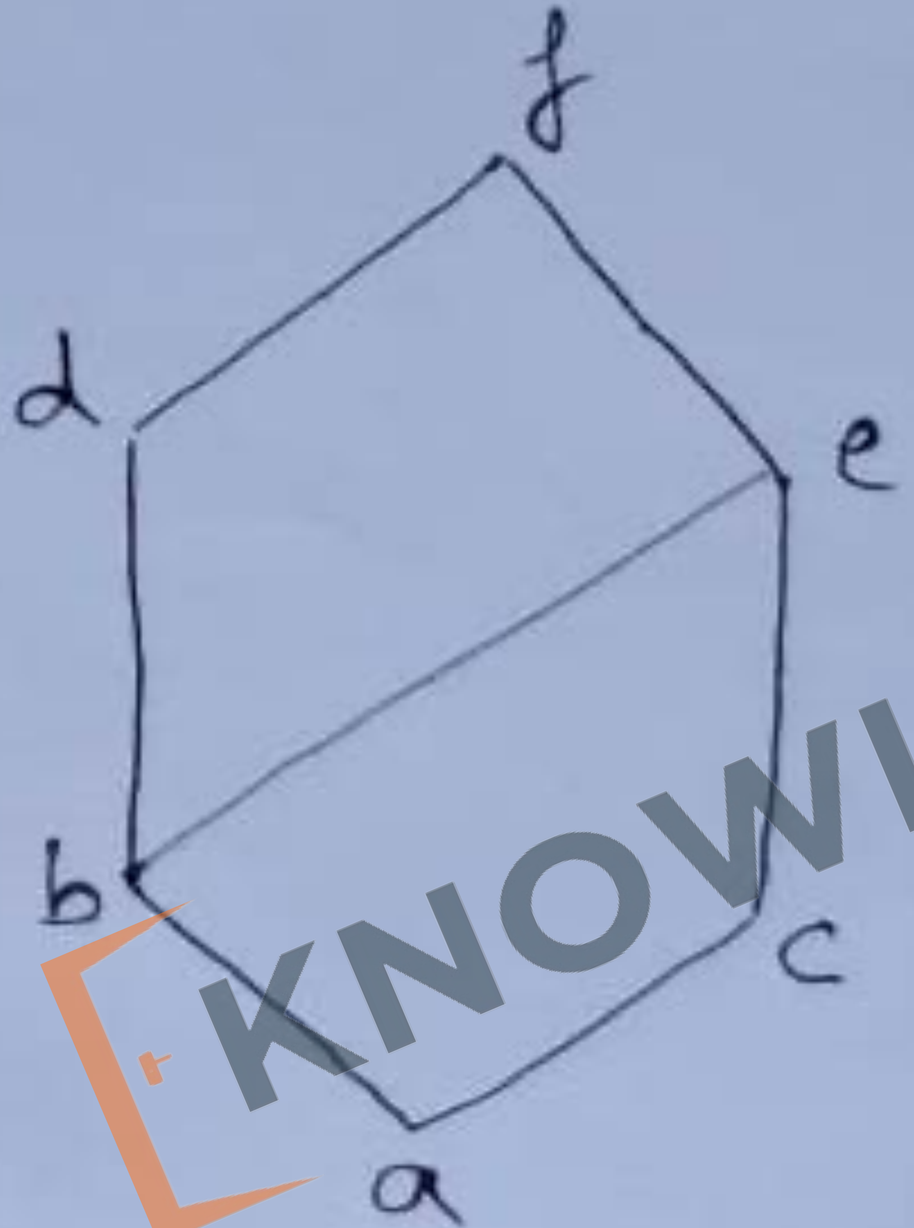
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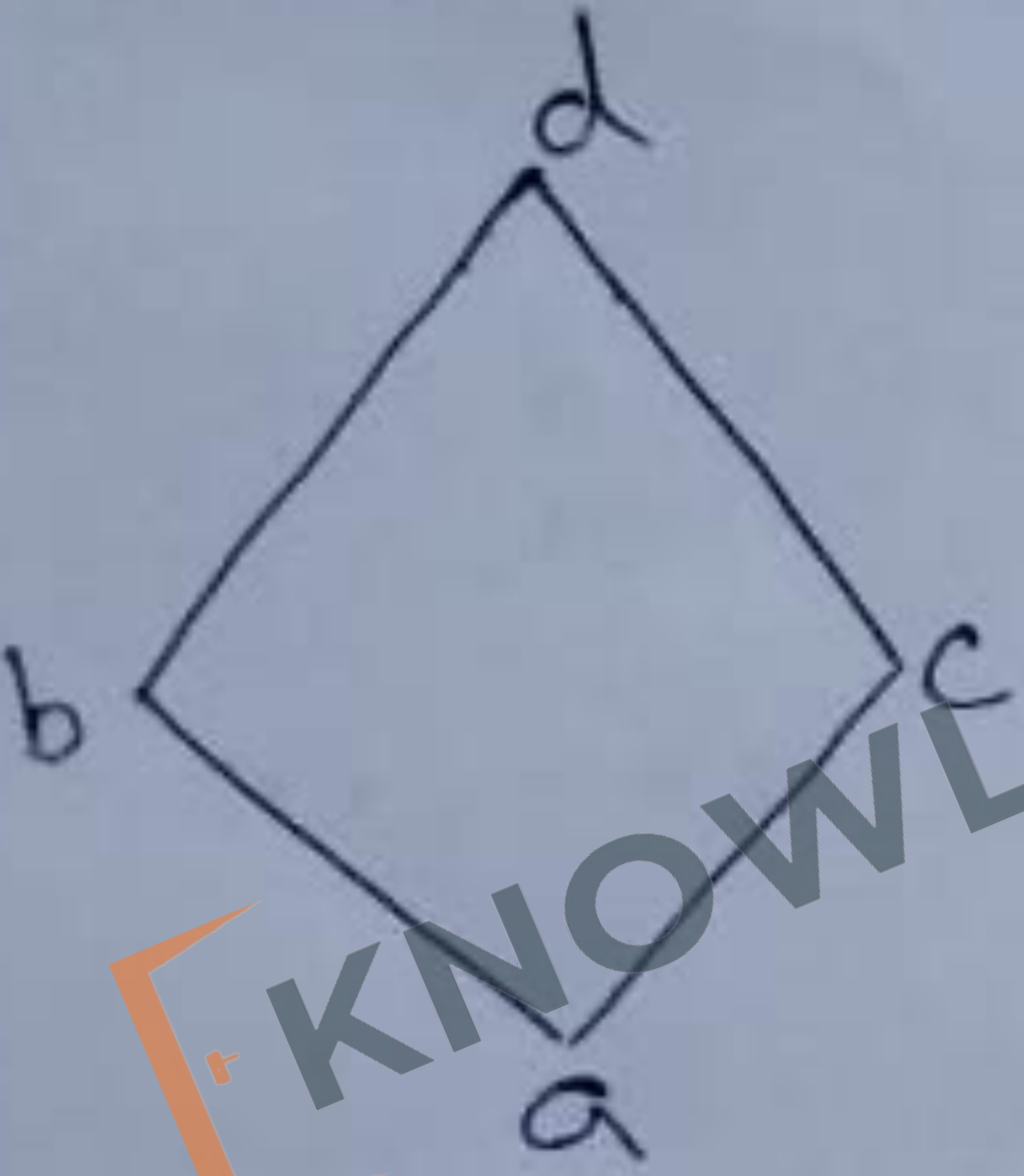
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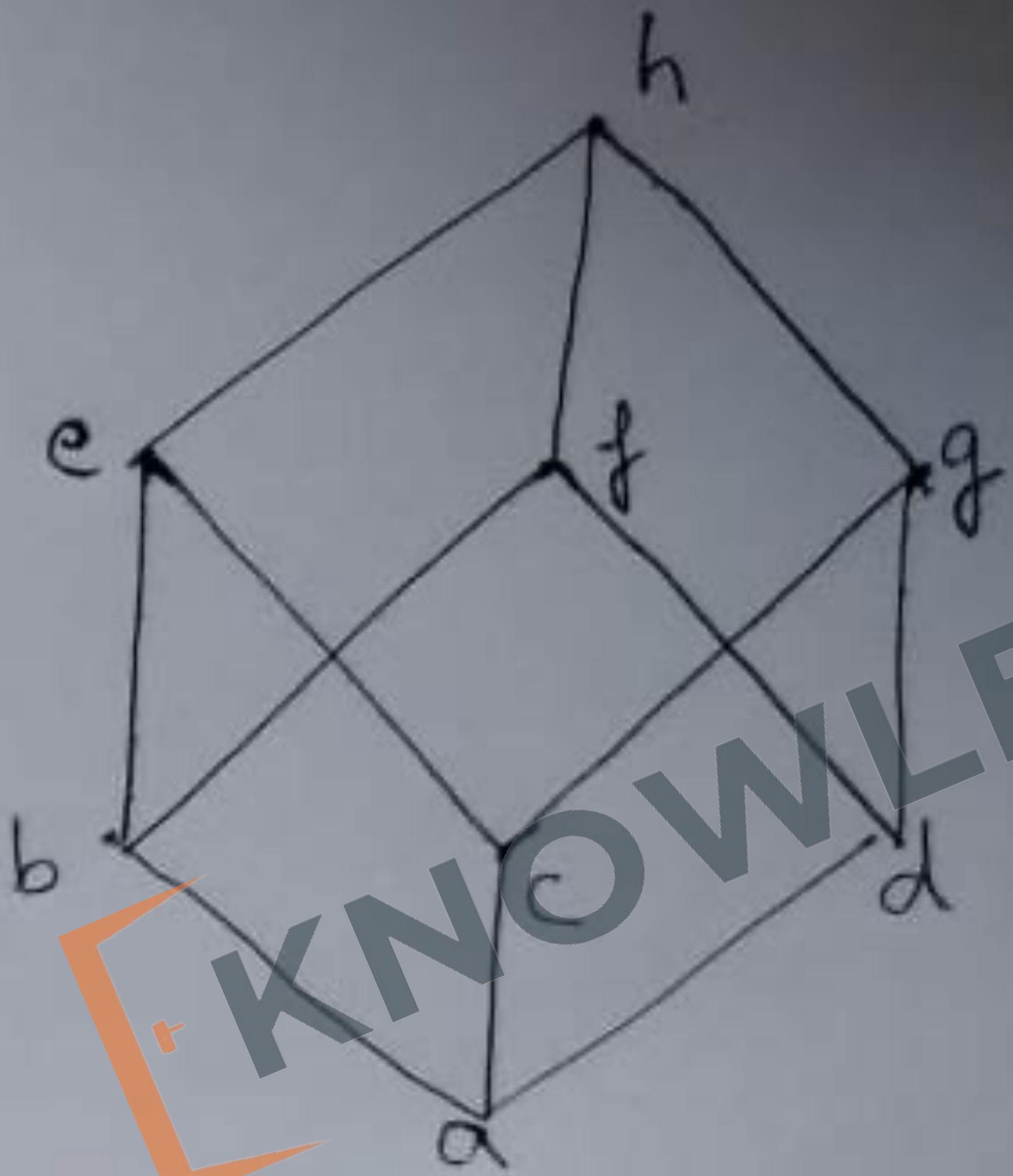
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Q The following is the Hasse diagram of the Poset [{a, b, c, d, e}, ≤]

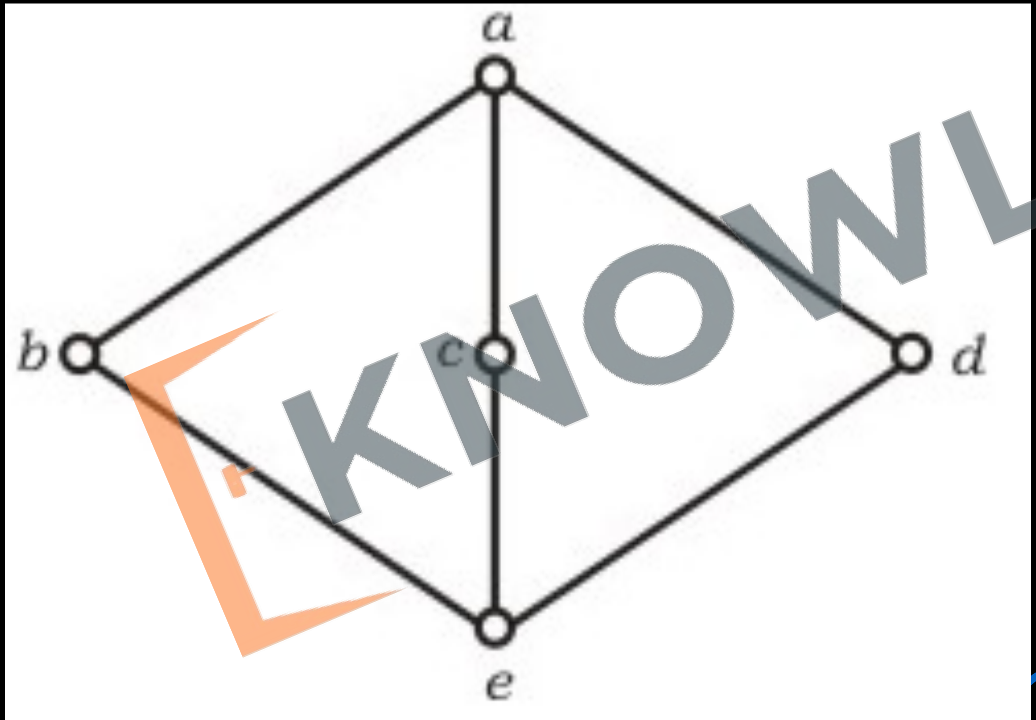
The Poset is

(A) not a lattice

(B) a lattice but not a distributive lattice

(C) a distributive lattice but not a Boolean algebra

(D) a Boolean algebra



Q Find which of the following is a lattice and Boolean Algebra?

(1) [{1,2,3,4,6,9}, /]

(2) [{2,3,4,6,12}, /]

(3) [{1,2,3,5,30}, /]

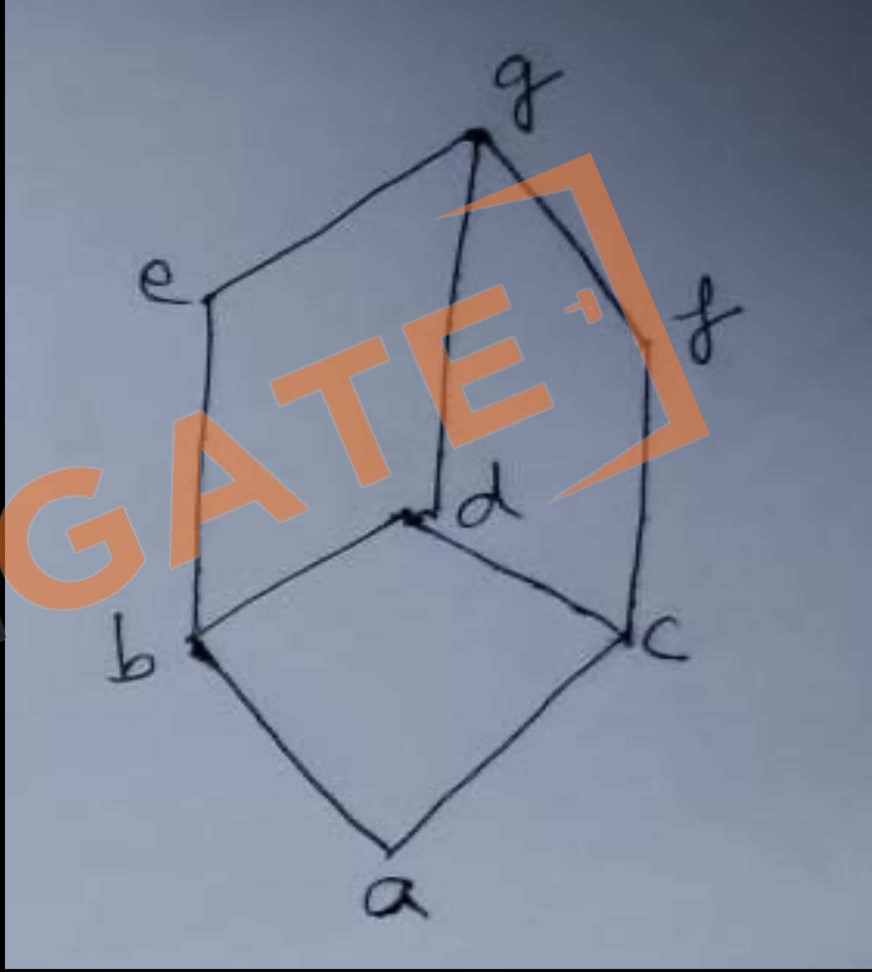
(4) [{1,2,3,6,9,18}, /]

(5) [{2,3,4,9,12,18}, /]

(6) [R, <=]

(7) [P(A), $\sqsubseteq$], A = {1,2,3}

Q Consider the following hasse diagram, find which of the following is true?



a) subset {a, b, c, g} is a lattice

b) subset {a, b, f, g} is a lattice

c) subset {a, d, e, g} is a lattice

d) subset {a, c, e, g} is a lattice

Chapter-4 (Functions)

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Function

- Functions are widely used in science, and in most fields of mathematics. It has been said that functions are "the central objects of investigation" in most fields of mathematics.

- Let X = {1, 2, 3, 4, 5} and Y = {2, 3, 4, 5, 6, 7} and R ⊑ X * Y. Now this is a valid relation but not a function, because there is a element which is not participating in the relationship secondly 5 is relating with more than one element.

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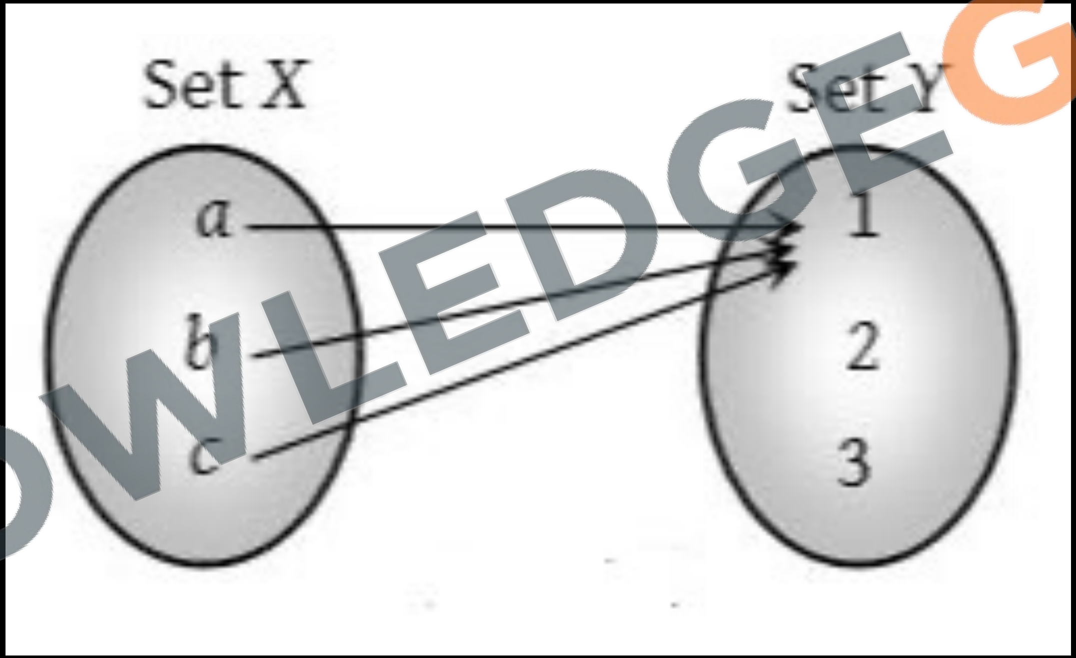
Function

- In mathematics, a function is a relation between sets that associates to every element of a first set exactly one element of the second set.
- A relation 'f' from a set 'A' to a Set 'B' is called a function, if each element of A is mapped with a unique element on B.
- f: A→B



- Range of fun ⊑ B
- Range of f = { y | y ϵ B and (x, y) ϵ f}

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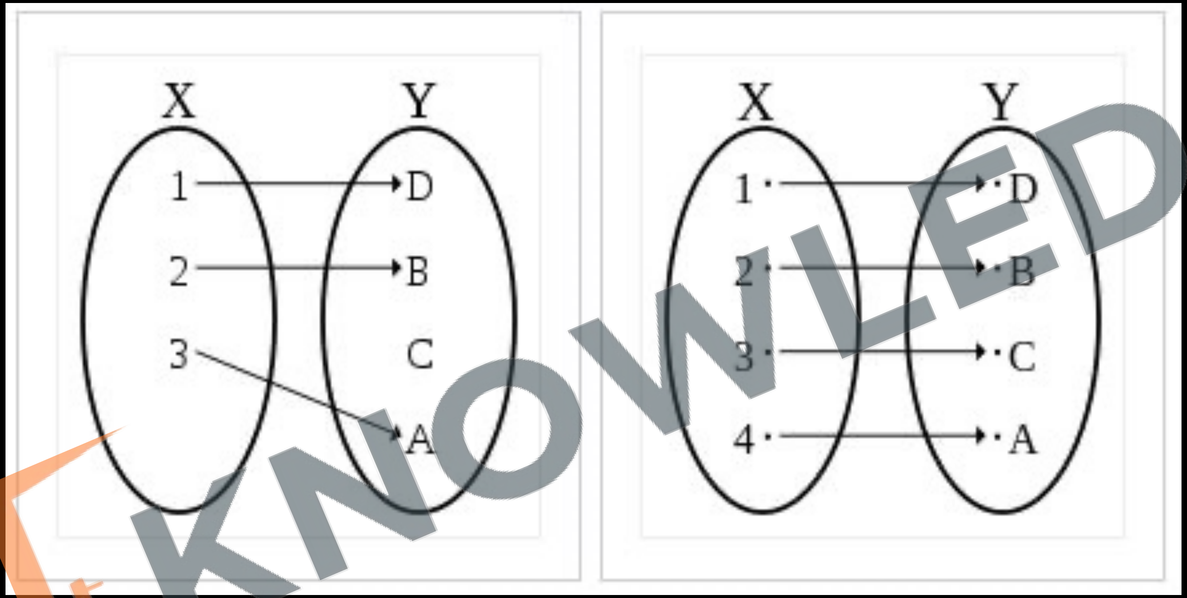
- If $|A| = m$ and $|B| = n$, then number of functions possible from A to B = ?

One-to-One (Injective Function)

- An **injective function** (also known as **injection**, or **one-to-one function**) is a function that maps distinct elements of its domain to distinct elements of its codomain. In other words, every element of the function's codomain is the image of *at most* one element of its domain.

One-to-One (Injective Function)

- A function F: A→B is said to be one-to-one function if every element of A has distinct image in B
- If A and B are finite set, then one-to-one from A→B is possible
  - if |A| <= |B|



- No of function possible = ?

- No of function possible = ${}^{n}p_{m}$ = P (n, m)
- If |A| = |B| = n, then no of functions possible is n!

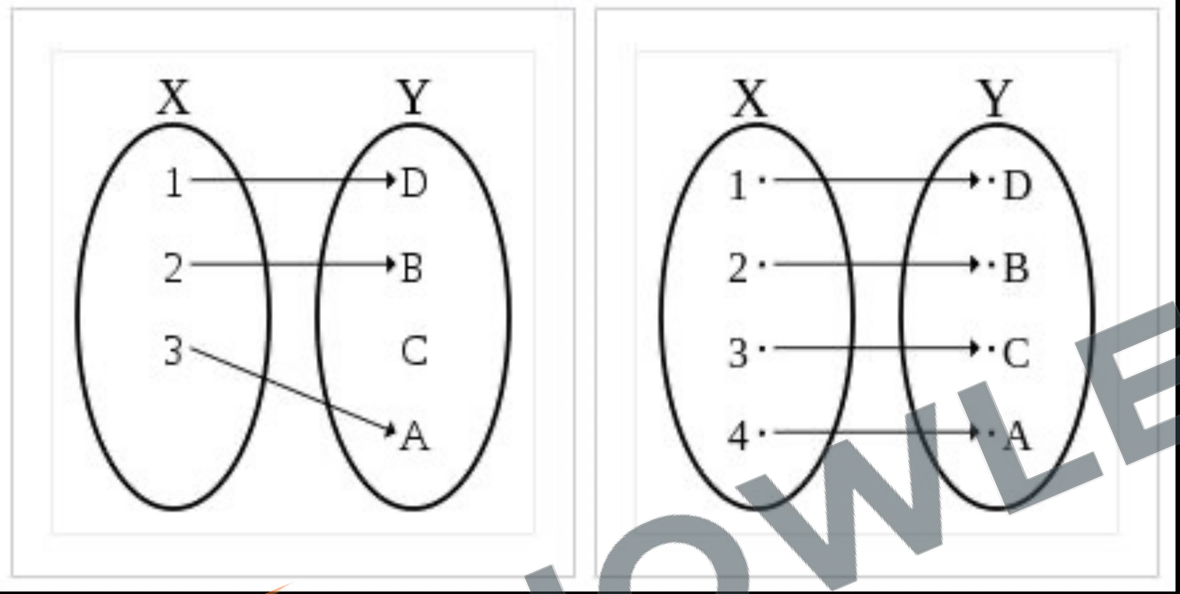
Q Let X and Y denote the sets containing 2 and 20 distinct objects respectively and F denote the set of all possible functions defined from X and Y. Let f be randomly chosen from F. The probability of f being one-to-one is _________

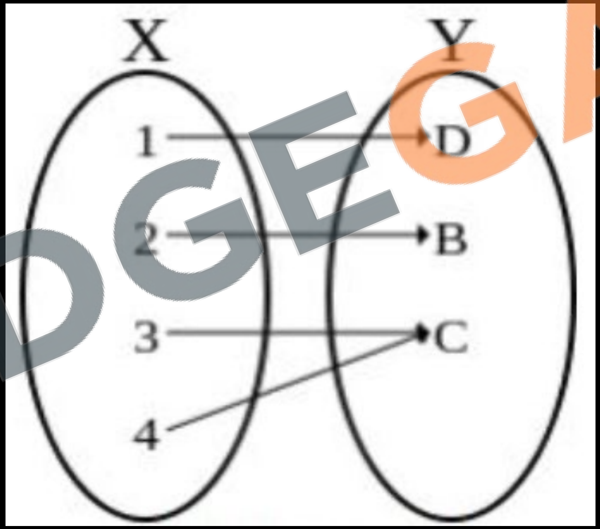
Onto (Surjective Function)

- A function f from a set X to a set Y is **surjective** (also known as **onto**, or a **surjection**), if for every element y in the co-domain Y of f, there is at least one element x in the domain X of f such that $f(x) = y$. It is not required that x be unique; the function f may map one or more elements of X to the same element of Y.

Onto (Surjective Function)

- A function f: A → B is said to be onto if and only if every element of B is mapped by at least one element of A.
- Range of f = B





- If A and B are finite sets, then onto function from A→B is possible, |B|<=|A|
- If |A| = |B|, then every onto function from A to B is also one-to-one function.

No of onto function possible from A to B

$= n^m - {}^nc_1(n-1)^m + {}^nc_2(n-2)^m - {}^nc_3(n-3)^m + \text{---------} + (-1)^n \, {}^nc_{n-1} \, 1^m$

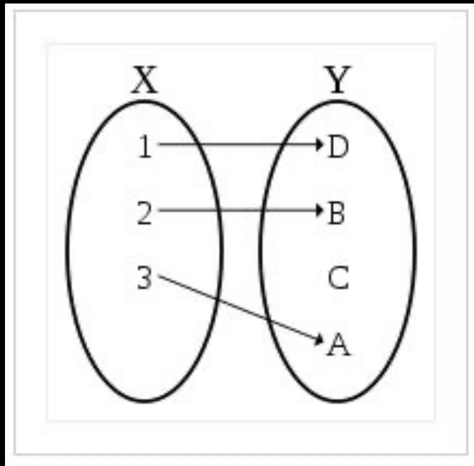
Q The number of onto functions (surjective functions) from set X = {1, 2, 3, 4} to set Y = {a, b, c} is ____________________

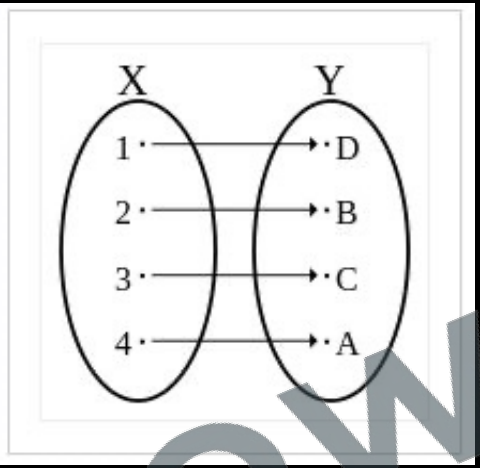
Q How many onto (or surjective) functions are there from an n-element (n >= 2) set to a 2-element set?

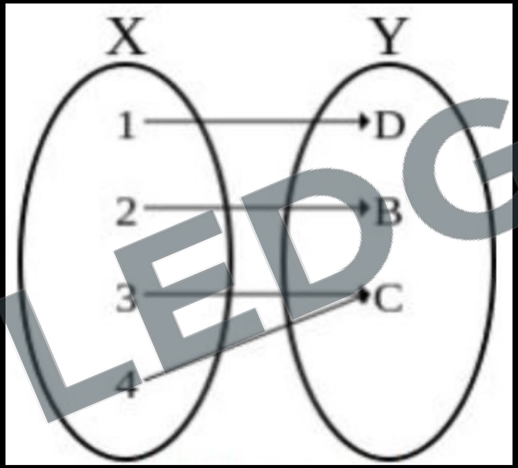
(A) 2^n **(B)** $2^n - 1$ **(C)** $2^n - 2$ **(D)** $2(2^n - 2)$

Bijective Function

- In mathematics, a **bijection**, **bijective function**, **one-to-one correspondence**, or **invertible function**, is a function between the elements of two sets, where each element of one set is paired with exactly one element of the other set, and each element of the other set is paired with exactly one element of the first set. There are no unpaired elements.



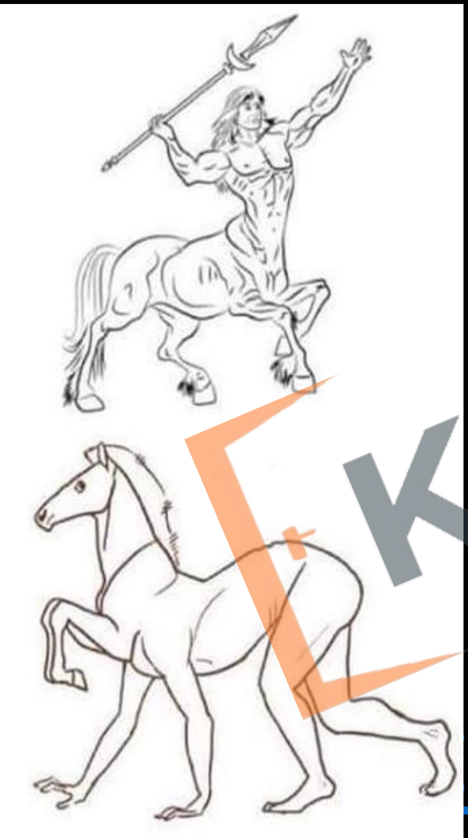


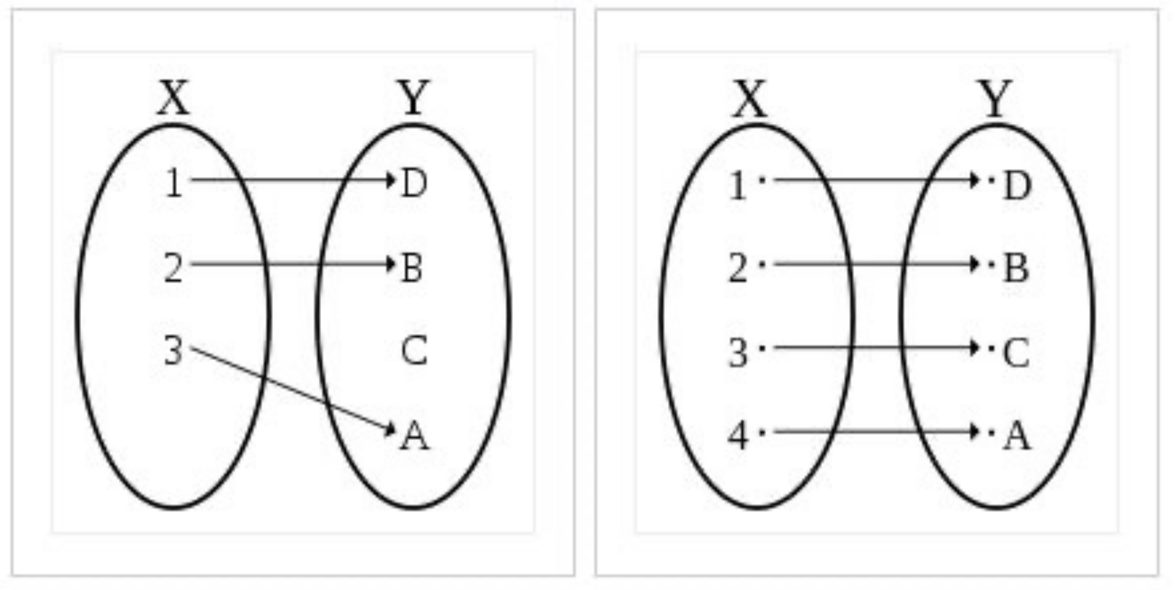


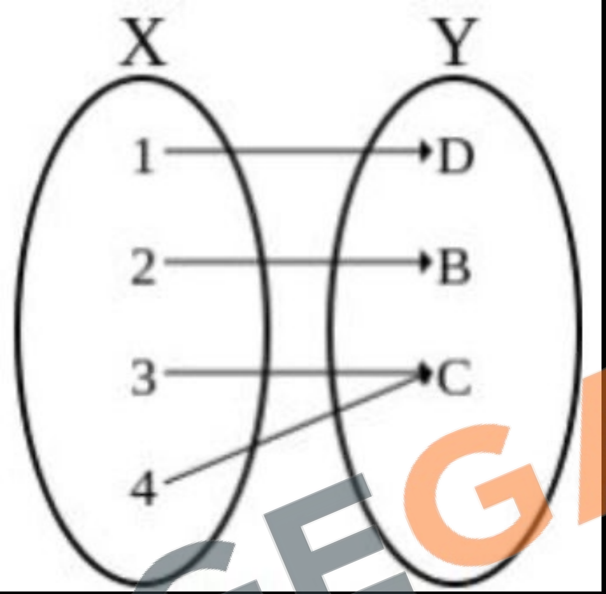
- A function f: A→B is said to be bijection if f is one-to-one and onto.
- Bijection from A and B is possible, if $|A| = |B|$
- No of Bijection from A to B = n!

Inverse of a function

- In mathematics, an **inverse function** (or **anti-function**) is a function that "reverses" another function
- If the function *f* applied to an input *x* gives a result of *y*, then applying its inverse function f^{-1} to *y* gives the result *x*, and vice versa.
- $f(x) = y$ then $f^{-1}(y) = x$.







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- $f(x) = 5x - 7$
- $f^{-1}(y) = (y + 7)/5$

Q Let *R* denote the set of real numbers. Let $f: R \times R \rightarrow R \times R$ be a bijective function defined by $f(x, y) = (x + y, x - y)$. The inverse function of *f* is given by

a) $f^{-1}(x, y) = (1 / (x + y), 1 / (x - y))$

b) $f^{-1}(x, y) = (x - y, x + y)$

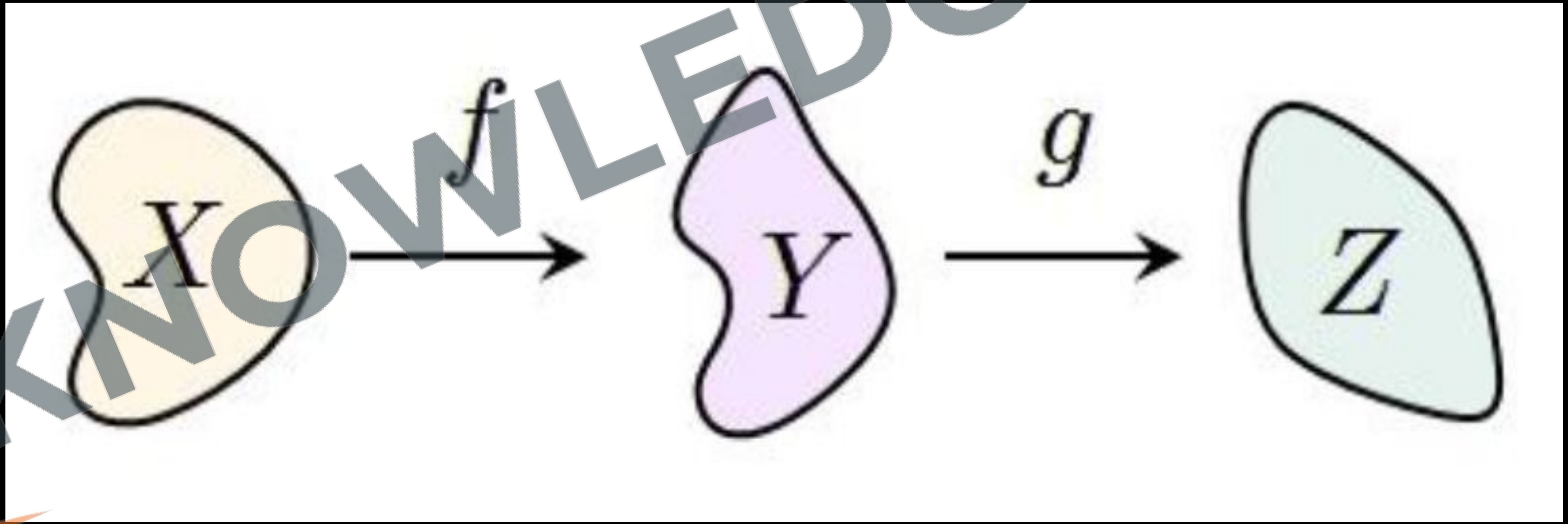
c) $f^{-1}(x, y) = ((x + y) / 2, (x - y) / 2)$

d) $f^{-1}(x, y) = [2(x - y), 2(x + y)]$

Function composition

- In mathematics, **function composition** is an operation that takes two functions f and g and produces a function h such that $h(x) = g(f(x))$.

- In this operation, the function g is applied to the result of applying the function f to x. That is, the functions $f: X \to Y$ and $g: Y \to Z$ are **composed** to yield a function that maps x in X to $g(f(x))$ in Z.



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- fog(x) = f(g(x))
- gof(x) = g(f(x))

- Composition of functions on a finite set: If f = {(1, 3), (2, 1), (3, 4), (4, 6)}, and g = {(1, 5), (2, 3), (3, 4), (4, 1), (5, 3), (6, 2)}, then $g \circ f$ = {(1, 4), (2, 5), (3, 1), (4, 2)}.
- The composition of functions is always associative—a property inherited from the composition of relations. That is, if f, g, and h are three functions with suitably chosen domains and codomains, then $f \circ (g \circ h) = (f \circ g) \circ h$

Q If $g(x) = 1-x$ and $h(x)=x / (x-1)$, then $g(h(x)) / h(g(x))$ is:

a) $h(x) / g(x)$ **b)** $-1 / x$ **c)** $g(x) / h(x)$ **d)** $x / (1-x)^2$

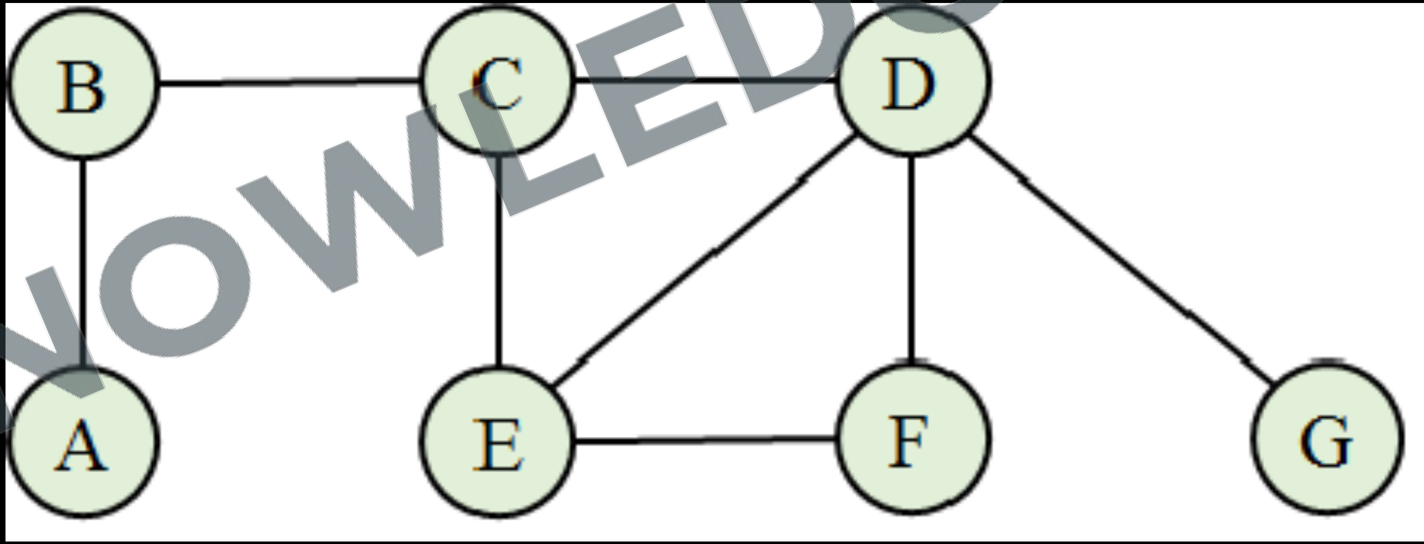
• Chapter-5 (Graph Theory)

Graph Theory

1. A graph G (V, E) consist of a set off objects $V = \{V_1, V_2, V_3....,V_N\}$ called vertices and another set $E = \{E_1, E_2, E_3,....,E_n\}$ whose elements are called edges.

2. Each edge e_k is identified with an unordered pair (v_i, v_j) of vertices.

3. The vertices v_i, v_j associated with edge e_k are called the end vertices of e_k.

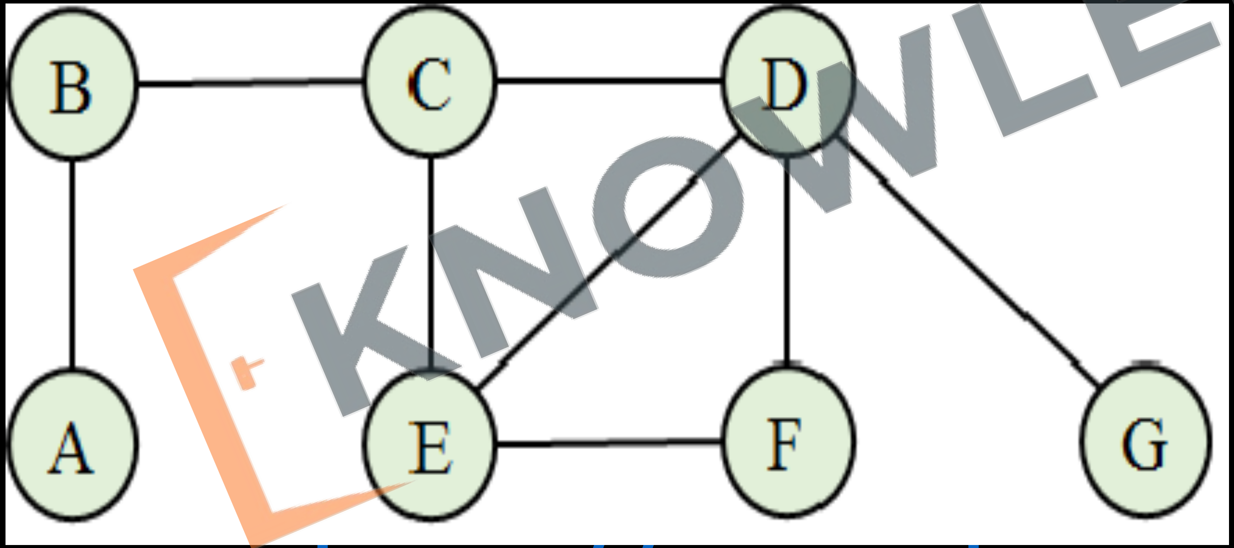


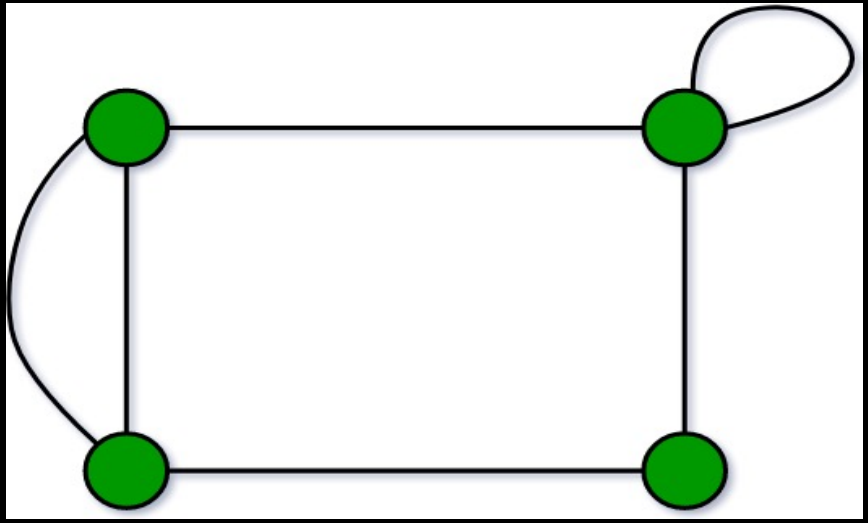
1. **Self-Loop:** Edge having the same vertex (v_i, v_i) as both its end vertices is called self-loop.

2. **Parallel Edge:** When more than one edge associated with a given pair of vertices such edges are referred as parallel edges.

3. **Adjacent Vertices:** If two vertices are joined by the same edges, they are called adjacent vertices.

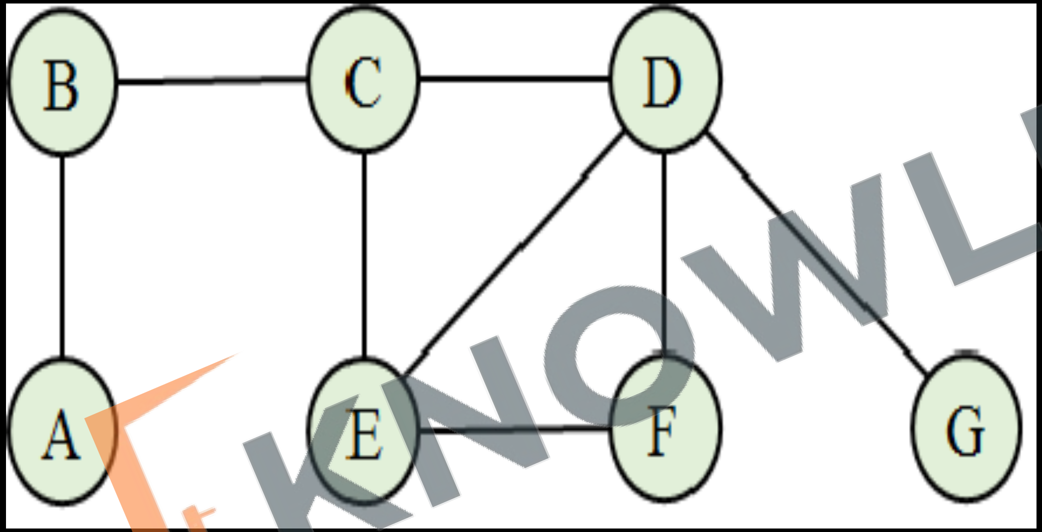
4. **Adjacent Edges:** If two edges are incident on some vertex, they are called adjacent edges.

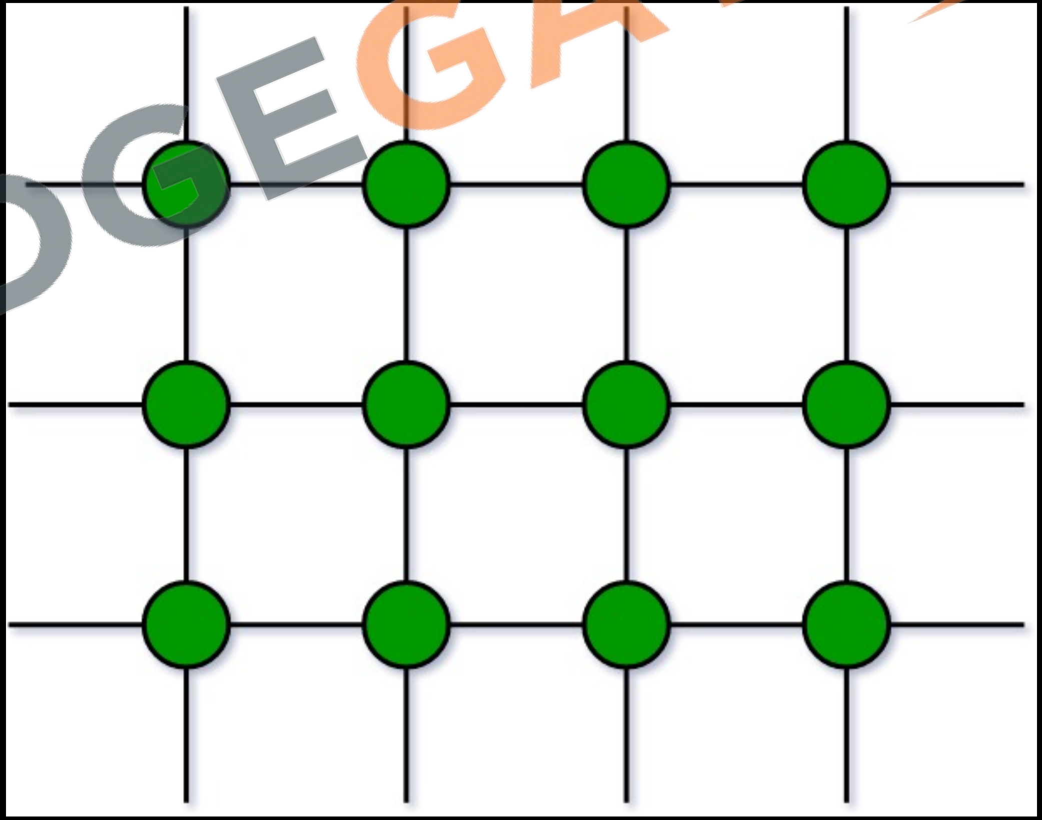




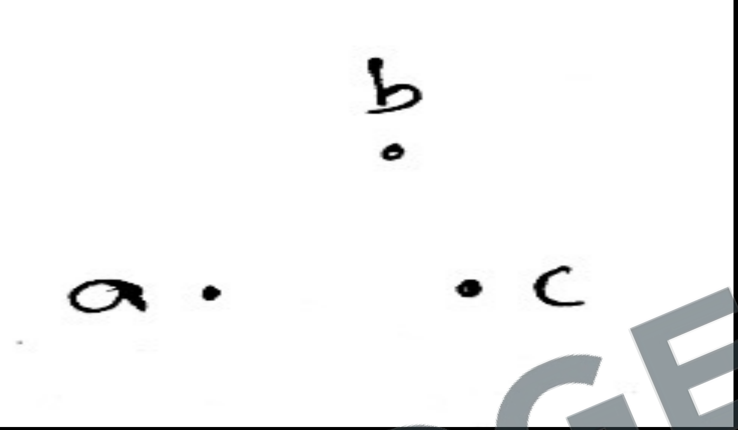
1. **Finite graph**: - A graph with finite number of vertices as well as the finite number of edges is called a finite graph.

2. For simple graph we can say if the number of vertices are finite then number of edges will also be finite.



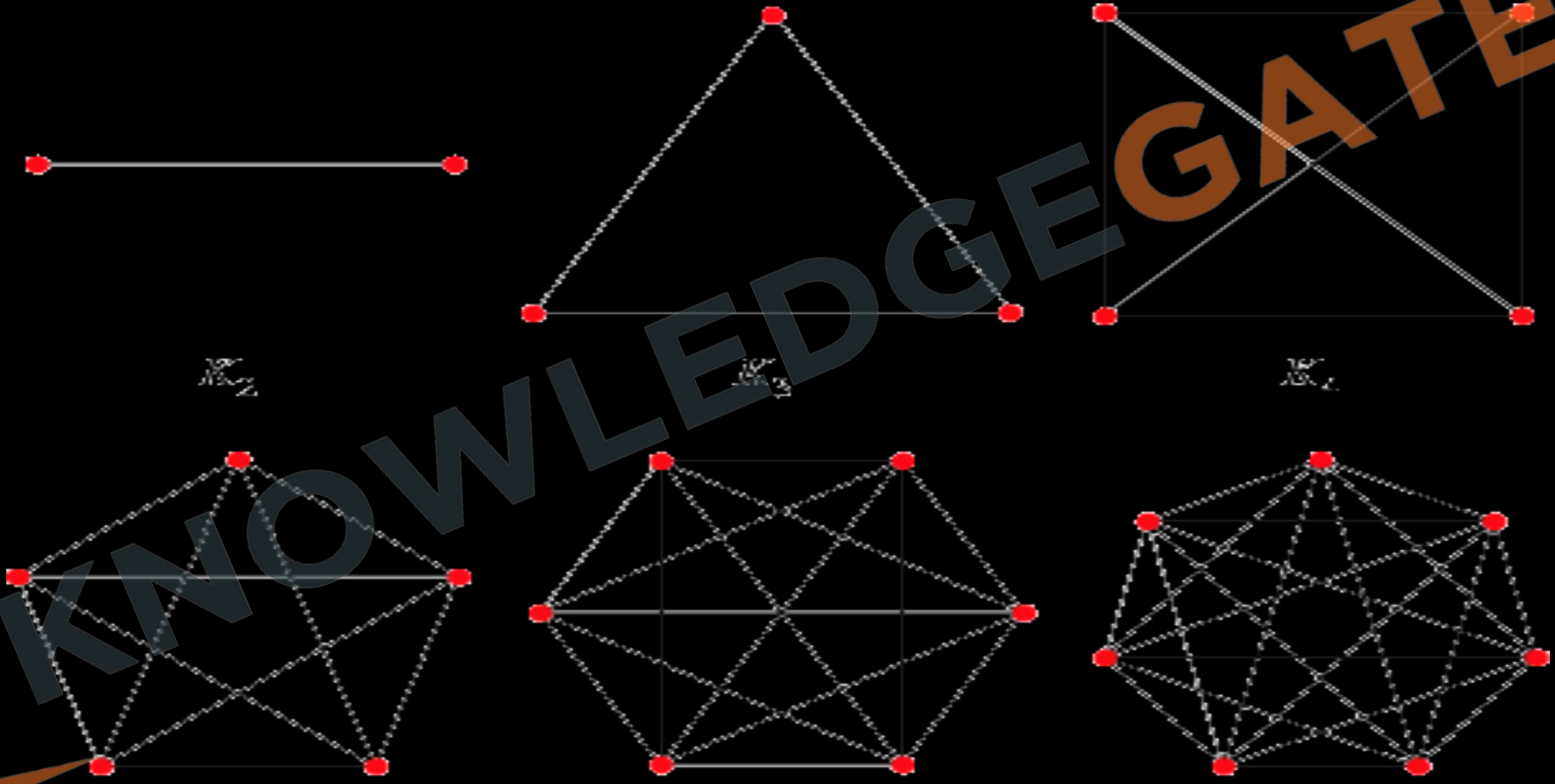


1. **Null Graph:** A graph is said to be null if edge set is empty E = {}, that is a graph with only vertices but no edges.



1. **Trivial Graph:** A graph with only one vertex without an edge is called trivial graph. It is the smallest possible.

- **Complete or Full Graph:** In a simple graph there exist an edge between each and every pair of vertices i.e. every vertex are adjacent to each other, then the graph is said to be a complete graph, denoted by K_n.



1. A simple graph with maximum number of edges are called Complete Graph.

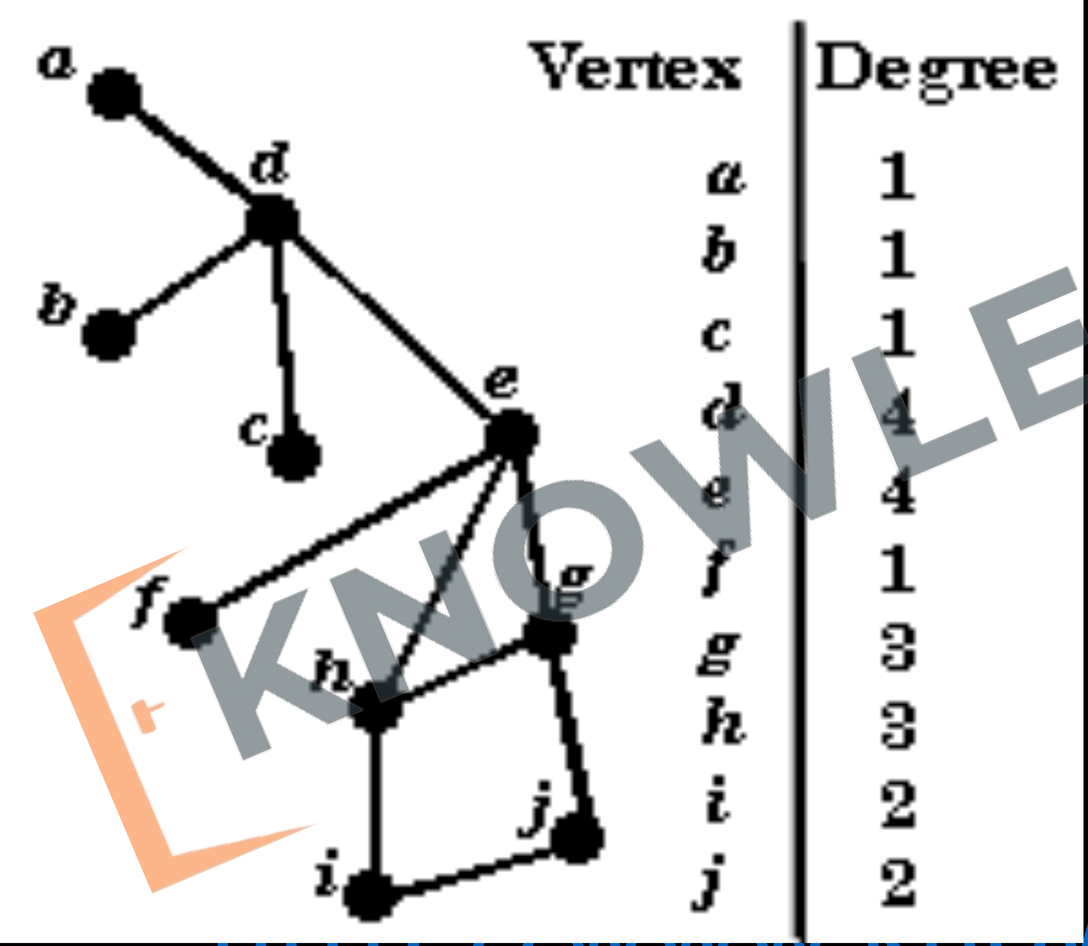
2. Number of edges in a simple graph is n(n-1)/2

Q Number of simple graph possible with n vertices?

Q Number of simple graph possible with n vertices and e edges?

Degree

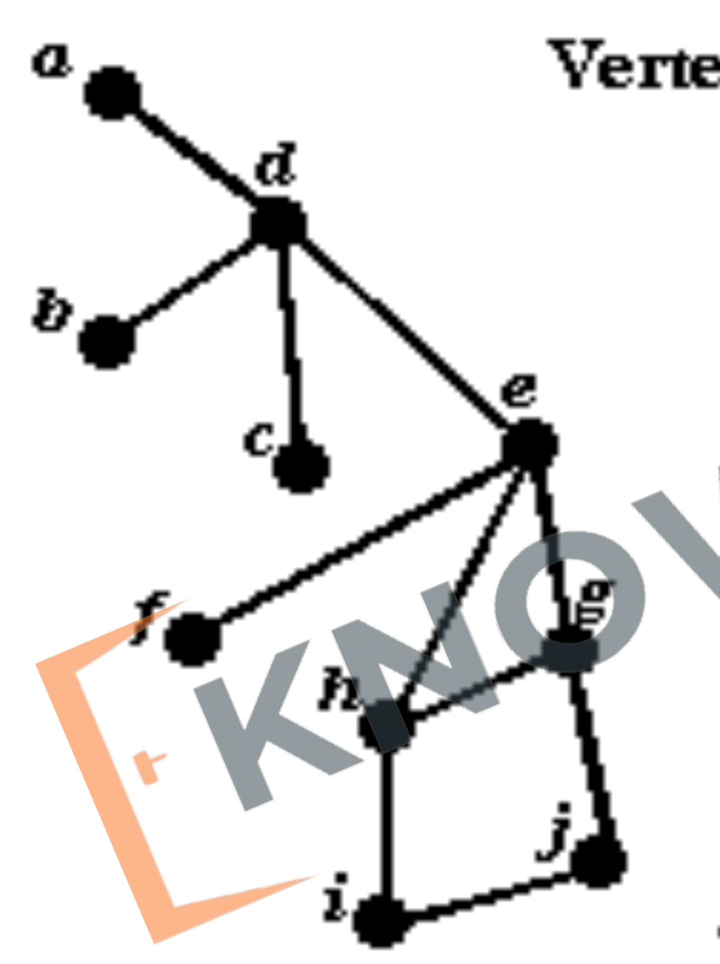
- **Degree of a Vertex:** The degree of a vertex in an undirected graph is the number of edges associated with it, denoted by deg(v_i).



Vertex	Degree
a	1
b	1
c	1
d	4
e	4
f	1
g	3
h	3
i	2
j	2

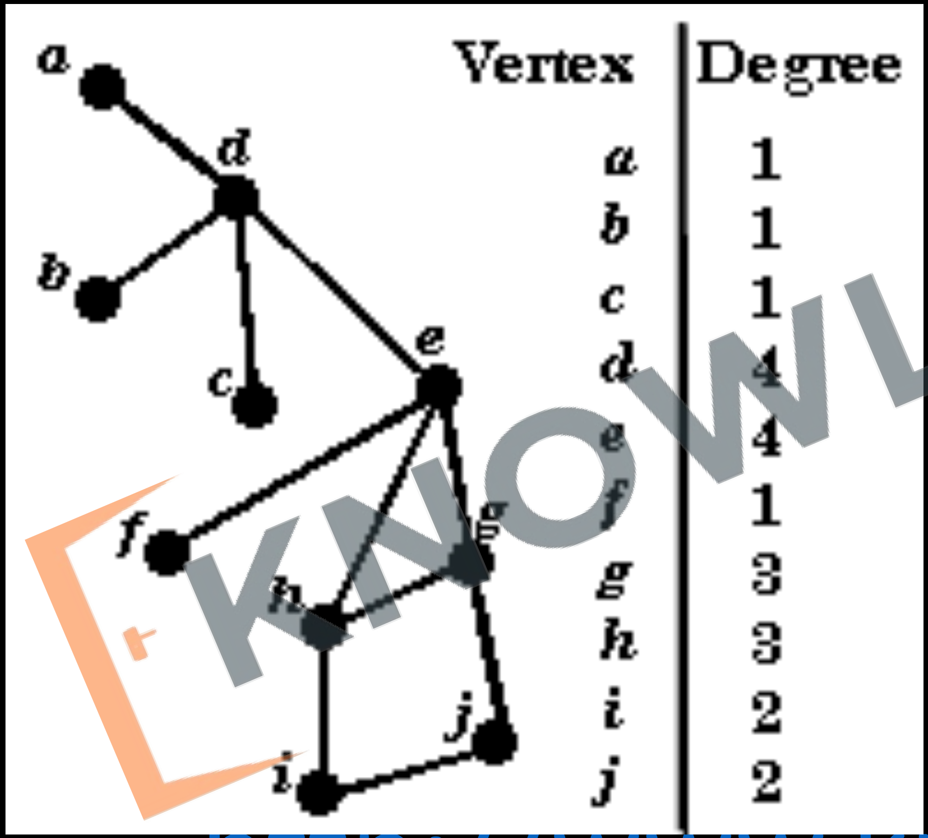
- **Isolated vertex:** A vertex with degree zero is called isolated vertex.

- **Pendant vertex:** A vertex with degree one is called pendant vertex.



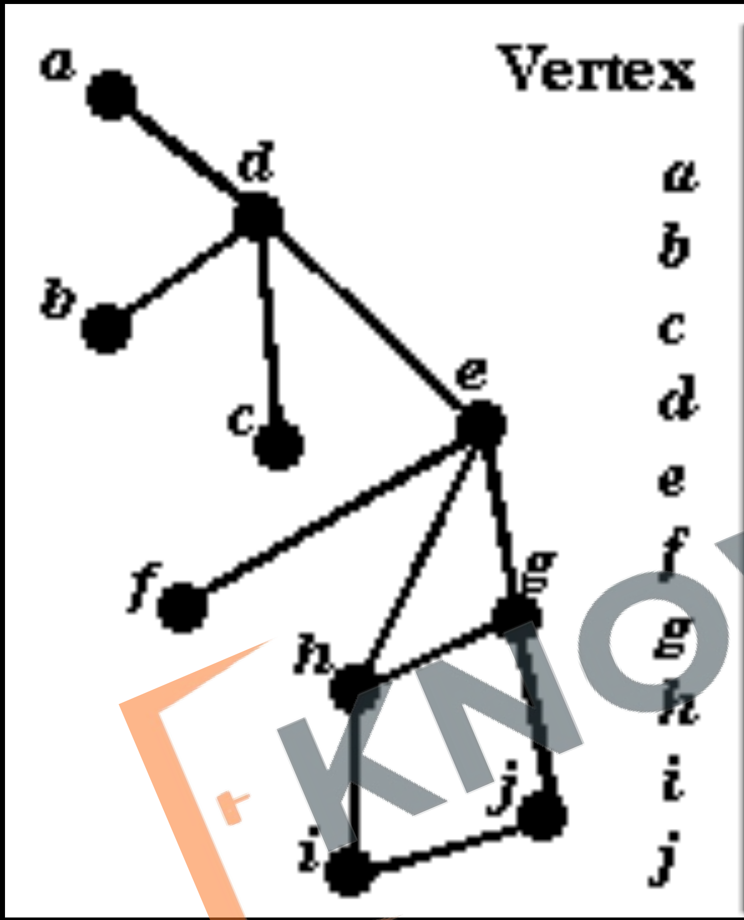
Vertex	Degree
a	1
b	1
c	1
d	4
e	4
f	1
g	3
h	3
i	2
j	2

- **Hand-shaking theorem:** - Since each edge contribute two degree in the graph, the sum of the degree of all vertices in G is twice the number of edges in g.
  - $\sum_{i=1}^{n} d(vi)$ = 2|E|



Vertex	Degree
a	1
b	1
c	1
d	4
e	4
f	1
g	3
h	3
i	2
j	2

- The number of vertices of odd degree in a graph is always even.
  - $\sum_{i=1}^{n} d(vi) = \sum_{even} d(vi) + \sum_{odd} d(vi)$



Vertex	Degree
a	1
b	1
c	1
d	4
e	4
f	1
g	3
h	3
i	2
j	2

Q A simple graph G contains 21 edges, 3 vertices of degree 4 and all the remaining vertices are of degree 2. Then number of vertices |v| is?

Q A simple non-directed graph G has 24 edges and degree of each vertex is 4, then find the |v|?

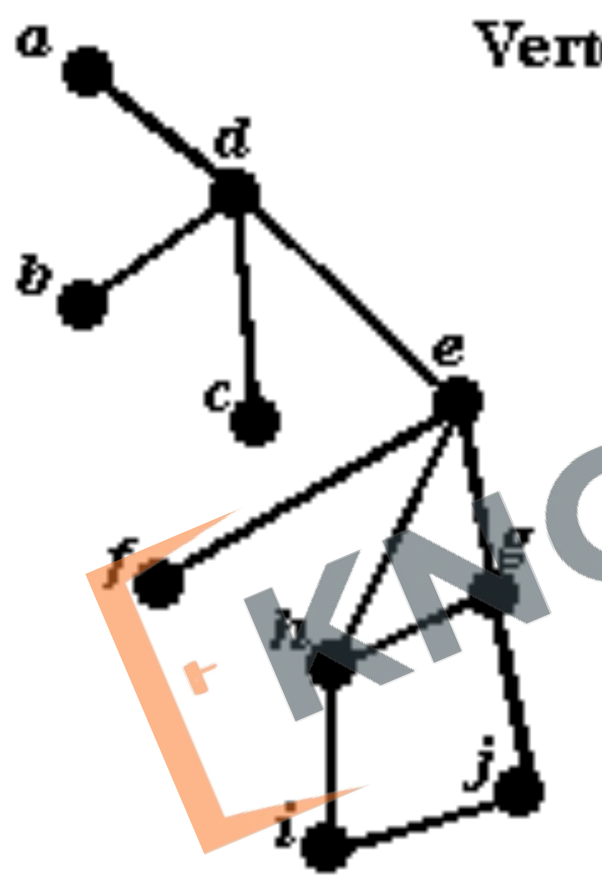
Q Consider a simple graph with 35 edges such that 4 vertex of degree 5, 5 vertex of degree 4, 4 vertex of degree 3, find the number of vertex with degree 2?

Q Simple non-directed graph G has 24 edges and degree of each vertex is K, the which of the following is possible no of vertices?

a) 20 **b)** 15 **c)** 10 **d)** 8

- δ(G) is the minimum possible degree of any vertex in a graph

- Δ(G) is the maximum possible degree of any vertex in a graph.



Vertex	Degree
a	1
b	1
c	1
d	4
e	4
f	1
g	3
h	3
i	2
j	2

$$\delta(G) * |V(G)| <= 2|E| <= \Delta(G) * |V(G)|$$

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Q G is undirected graph with n vertices and 25 edges such that each vertex has degree at least 3. Then the maximum possible value of n is

____

Q Minimum number of vertices possible in a simple graph if 41 edges and degree of each vertex is at most 5?

1. The **Havel–Hakimi algorithm** is an algorithm in graph theory solving the graph realization problem. That is, it answers the following question: Given a finite list of nonnegative integers, is there a simple graph such that its degree sequence is exactly this list.

2. Here, the "degree sequence" is a list of numbers that for each vertex of the graph states how many neighbors it has. For a positive answer the list of integers is called *graphic*.

3. The algorithm constructs a special solution if one exists or proves that one cannot find a positive answer. This construction is based on a recursive algorithm. The algorithm was published by Havel (1955), and later by Hakimi (1962).

Q Which of the following degree sequence represent a simple non-directed graph?

1) {2, 3, 3, 4, 4, 5}

2) {2, 3, 4, 4, 5}

3) {3, 3, 3, 1}

4) {1, 3, 3, 4, 5, 6, 6}

5) {2, 3, 3, 3, 3}

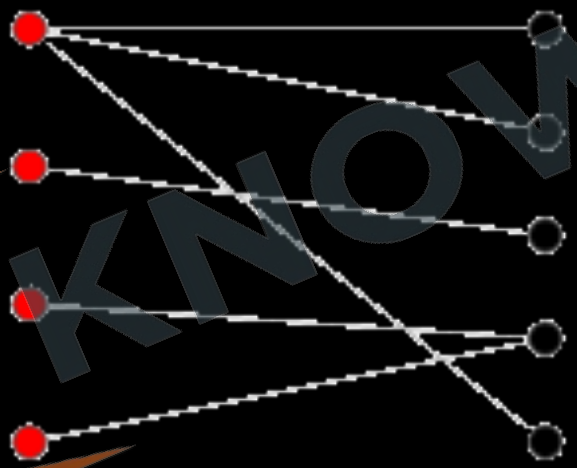
6) {6, 6, 6, 6, 4, 3, 3, 0}

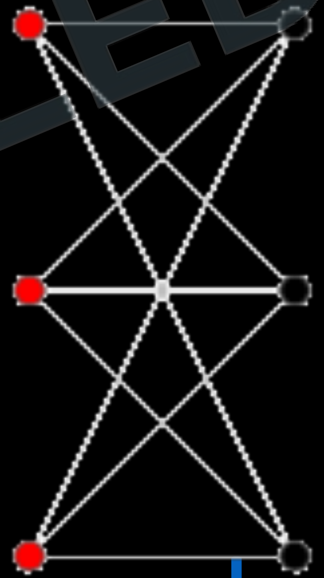
7) {6, 5, 5, 4, 3, 3, 2, 2, 2}

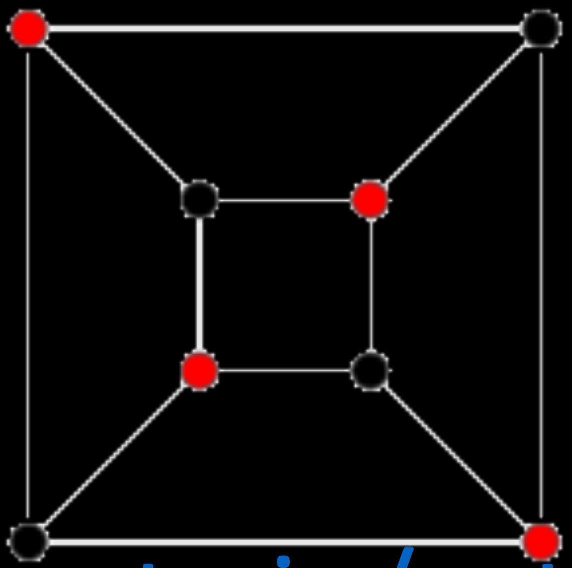
Some Popular Graph

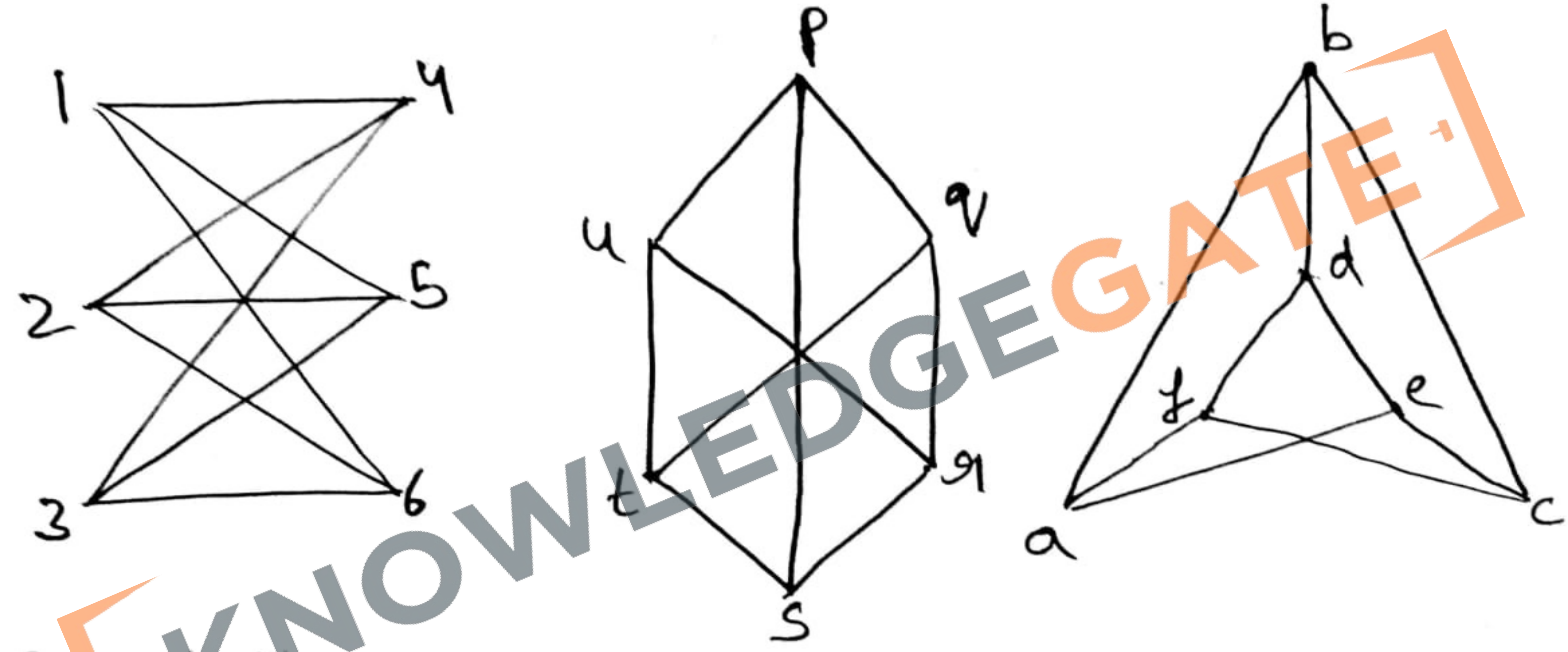
1. **Bi-partite graph:** - A graph G(V, E) is called bi-partite graph if it's vertex set V(G) can be partitioned into two non-empty disjoint subset $V_1(G)$ and $V_2(G)$ in such a way that each edge e ϵ E(G) has it's one end point in $V_1(g)$ and other end point in $V_2(g)$. The partition $V = V_1 \cup V_2$ is called bipartition of G.

2. **Complete Bi-partite graph:** - A Bi-partite graph G(V, E) is called Complete bi-partite graph if every vertex in the first partition is connected to every vertex in the second partition, denoted by $K_{m,n}$.









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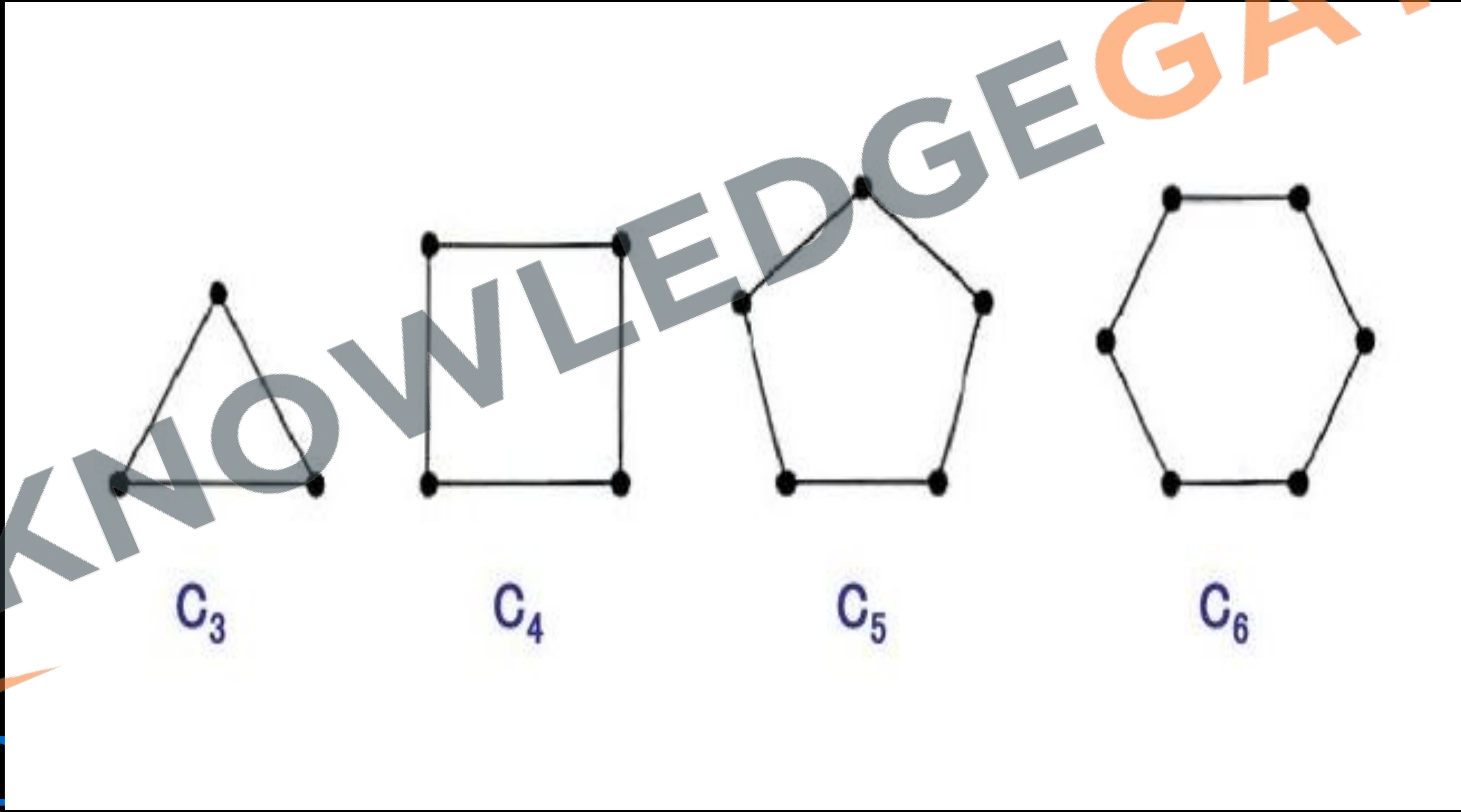
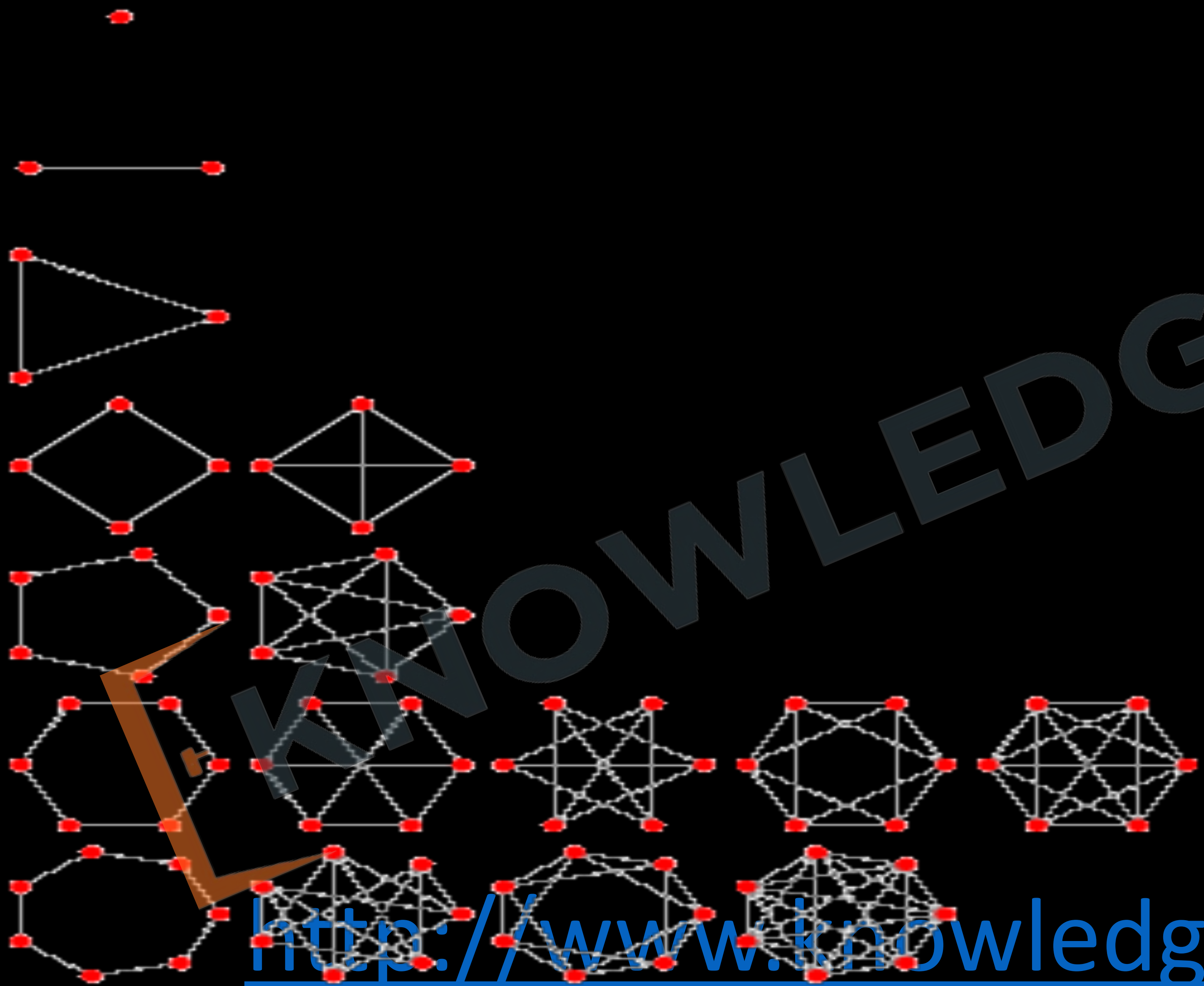
- **Cycle Graph:** - A cycle graph or circular graph is a graph that consists of a single cycle, or in other words, some number of vertices (at least 3) connected in a closed chain. The cycle graph with *n* vertices is called C_n. The number of vertices in C_n equals the number of edges, and every vertex has degree 2; that is, every vertex has exactly two edges incident with it.

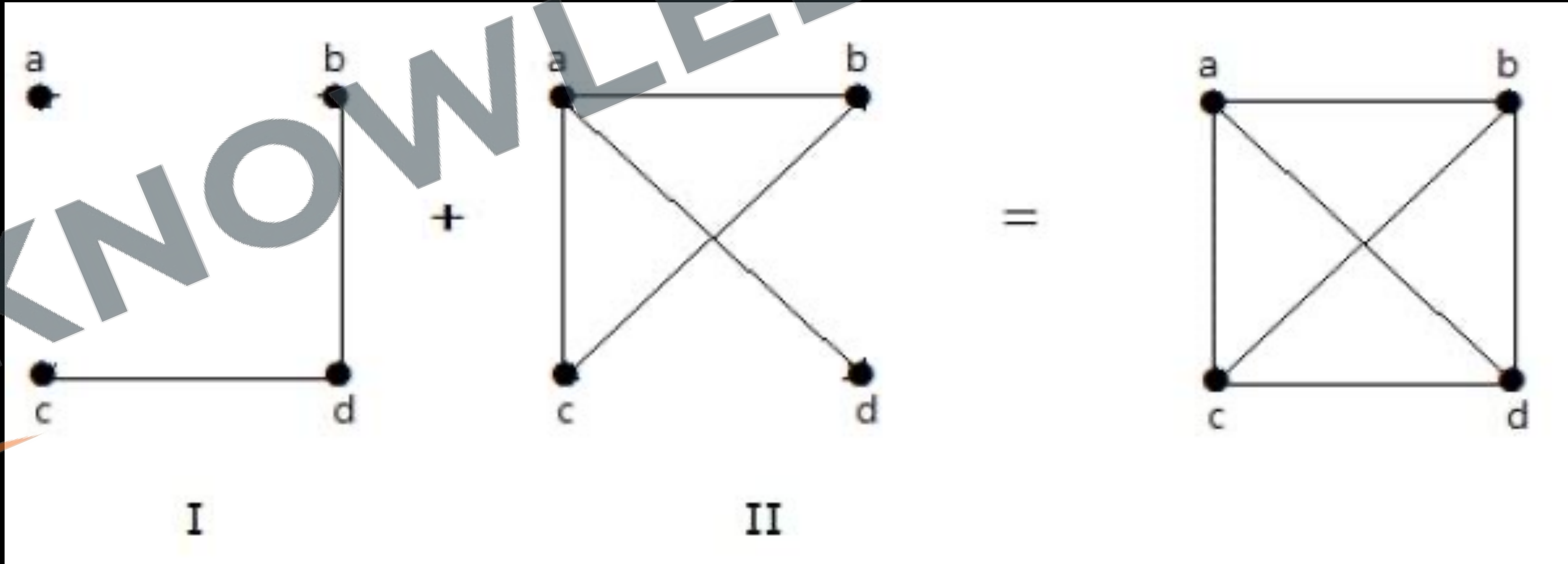


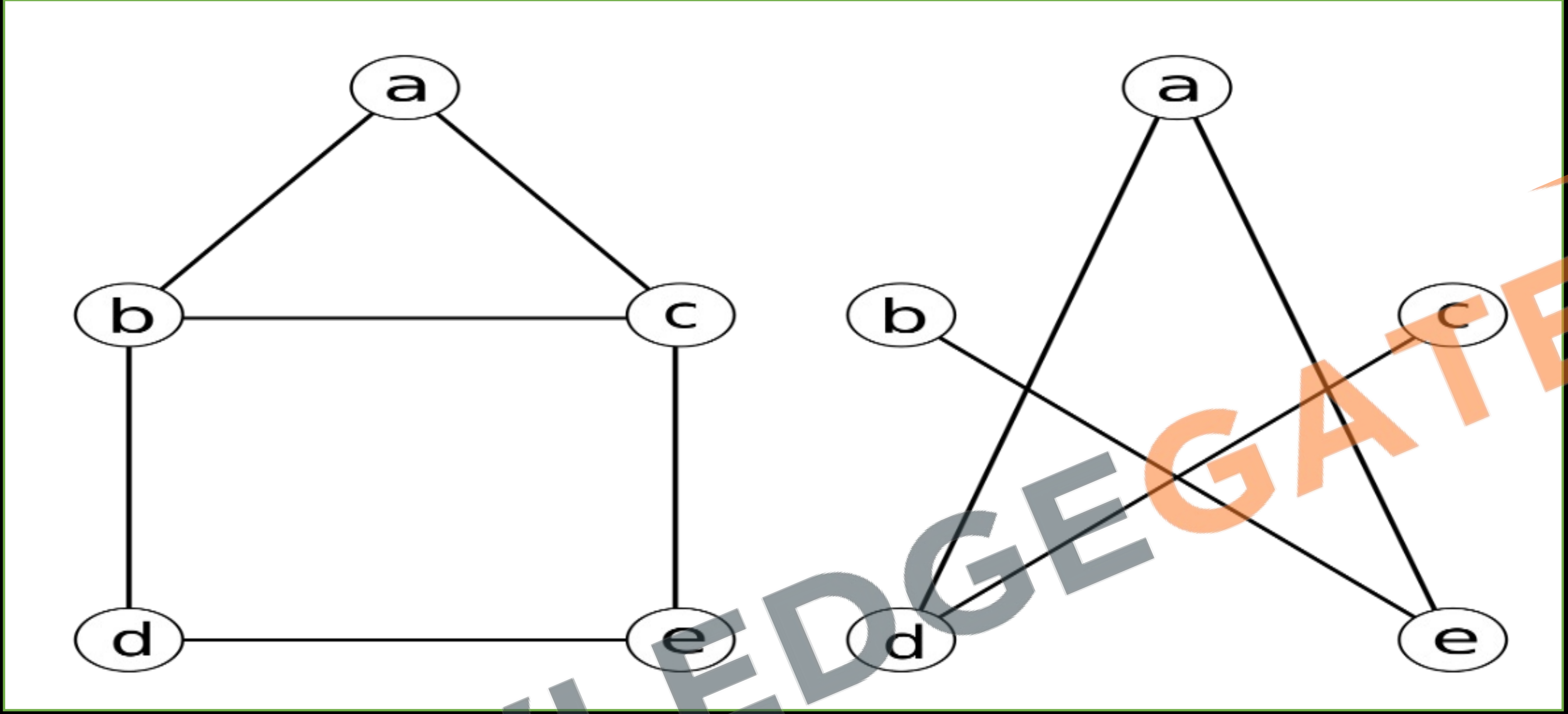
- **Regular graph:** - A graph in which all the vertices are of equal degree is called a regular graph. E.g. 2-regular graph, 3-regular graph.



Complement of a Graph

1. The complement of a simple graph G (V, E) is a graph G^c (V, E^c) on the same vertices set as of G, such that there will be an edge between two vertices u, v in G^c if ang only if there is no edge between u, v in G. i.e. two vertices of Gc are adjacent iff they are not adjacent in G.
2. $V(G) = V(G^c)$
3. $E(G^c) = \{(u, v) \mid (u, v) \notin E(G)\}$
4. $E(G^c) = E(K_n) - E(G)$




a
b
c
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e
a
b
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e

Properties

1. $G \cup G^c = K_n$

2. $G \cap G^c$ = null graph

3. $|E(G)| + |E(G^c)| = E(K_n) = n(n-1)/2$

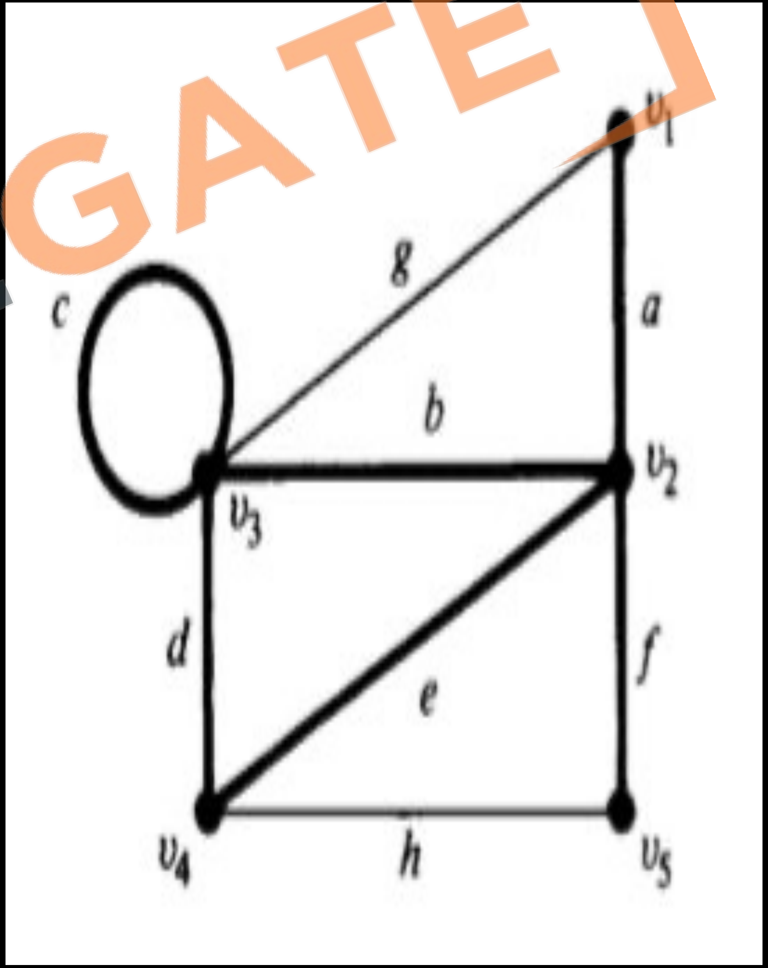
Q A simple graph G has 30 edges and G^c has 36 edges, the number of vertices in G will be?

Q A simple graph G has 56 edges and G^c has 80 edges, the number of vertices in G will be?

Q A simple graph G has $|v|=8$ and $|E|=12$, find number of edges in $|E(G^c)|$?

Traversal

- **Walk / Edge Train / Chain**: -A Walk is defined as a finite alternating sequence of vertices and edges, beginning and ending with vertices, such that each edge is incident with the vertices preceding and following it. Both vertex and edges may appear more than once.
- An **open walk** in graph theory is a walk that starts and ends at different vertices, whereas a **closed walk** starts and ends at the same vertex.
- An open walk becomes a path when it does not revisit any vertices Number of edges in a path is called length of a path.



Traversal	Walk	Open Walk	Closed Walk	Path
V_1 g V_3 b V_2 e V_4 d V_3 b V_2				
V_1 a V_2 e V_4 d V_3 b V_2 f V_5				
V_1 g V_3 c V_3 b V_2 a V_1				
V_1 a V_2 b V_3 d V_4 h V_5				

- **Connected Graph:** A graph is said to be connected if there is at least one path between every pair of vertices in G.
- A graph with n vertices can be connected with minimum n -1 edges.
- A graph with n vertices will necessary be connected if it has more than (n - 1) (n - 2)/2 edges.
- if a graph (connected or disconnected) has exactly two vertices of odd degree, there must be a path joining these two vertices

Q Which condition is necessarily for a graph to be connected?

a) A graph with 6 vertices and 10 edges

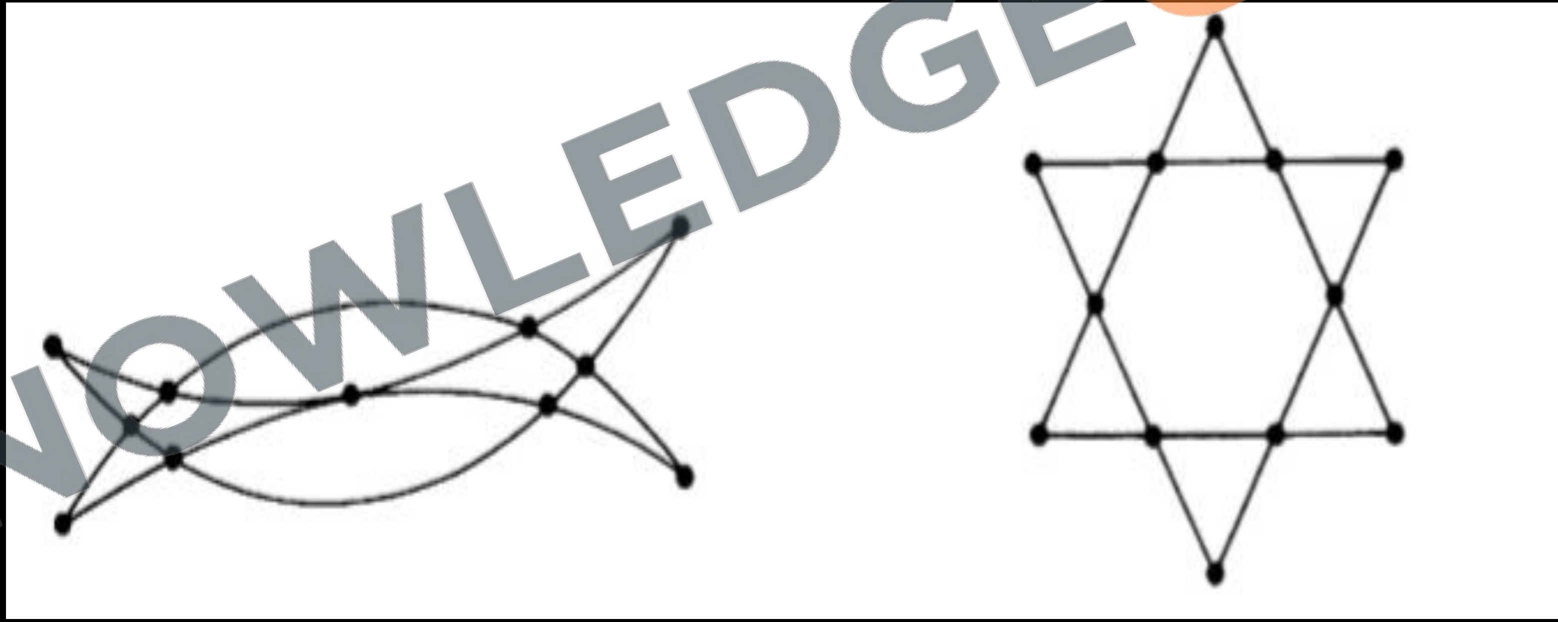
b) A graph with 7 vertices and 14 edges

c) A graph with 8 vertices and 22 edges

d) A graph with 9 vertices and 28 edges

Euler Graph

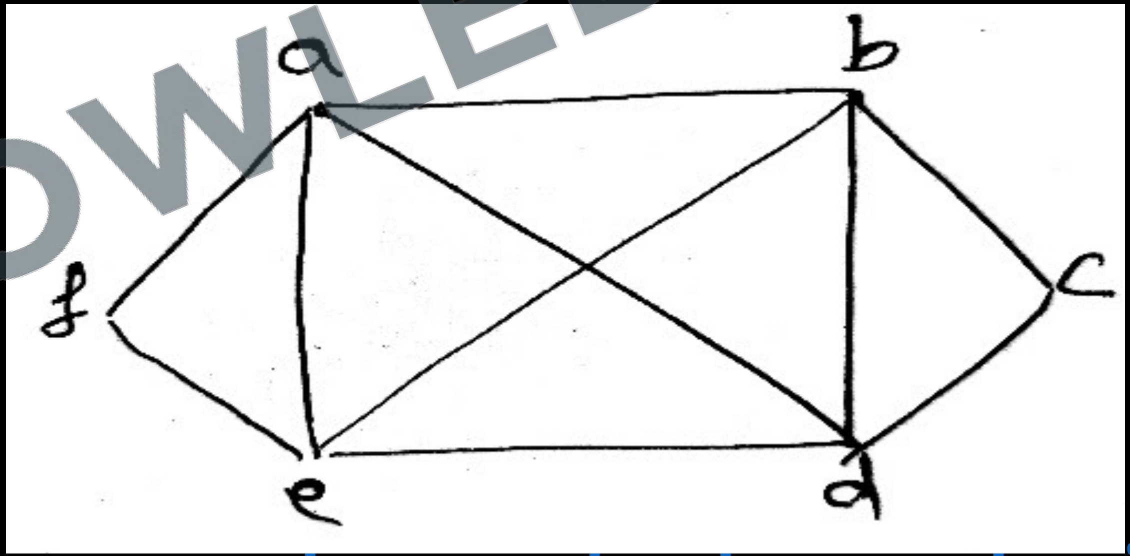
- **Euler Graph:** - If some closed walk in a graph contains all the edges of the graph(connected), then the walk is called a Euler line and the graph a Euler Graph.
- A given connected graph G is a Euler graph if and only if all vertices of G are of even degree.

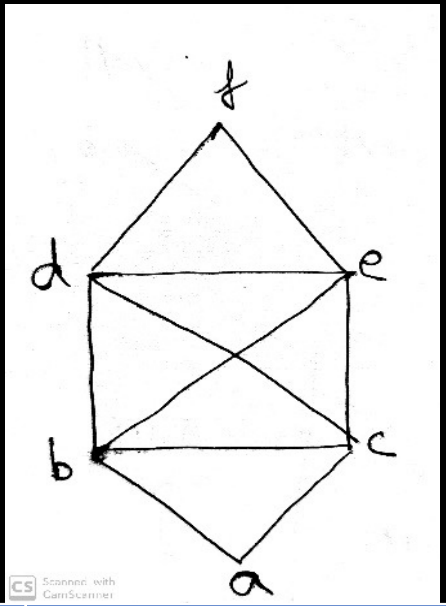


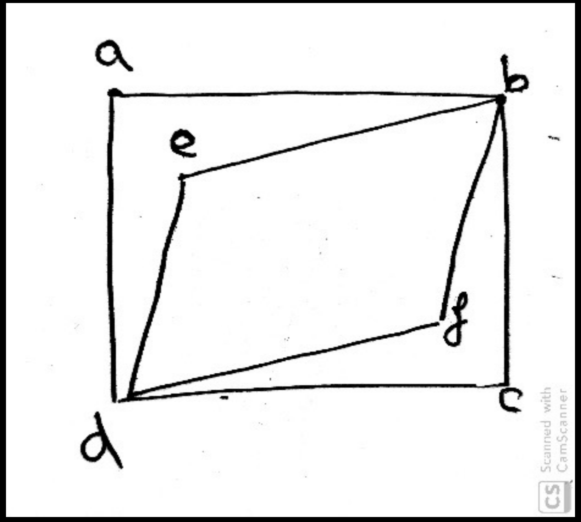
Hamiltonian

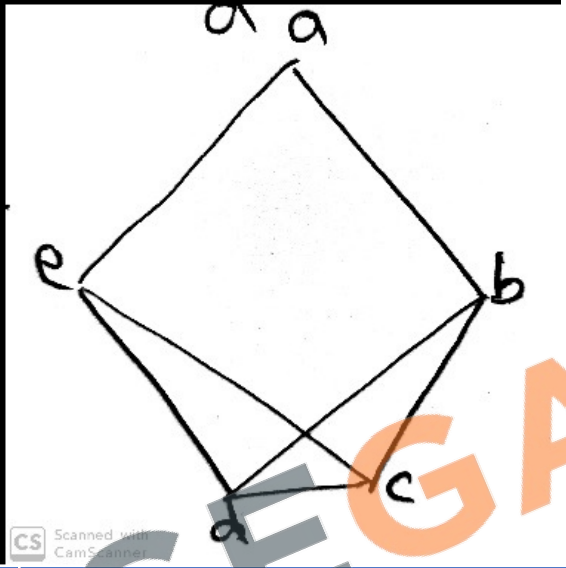
1. **Hamiltonian Graph:** - A Hamiltonian circuit in a connected graph is defined as a closed walk that traverses every vertex of G exactly once, except of course the starting vertex, at which the walk also terminates. A graph containing Hamiltonian circuit is called Hamiltonian graph.

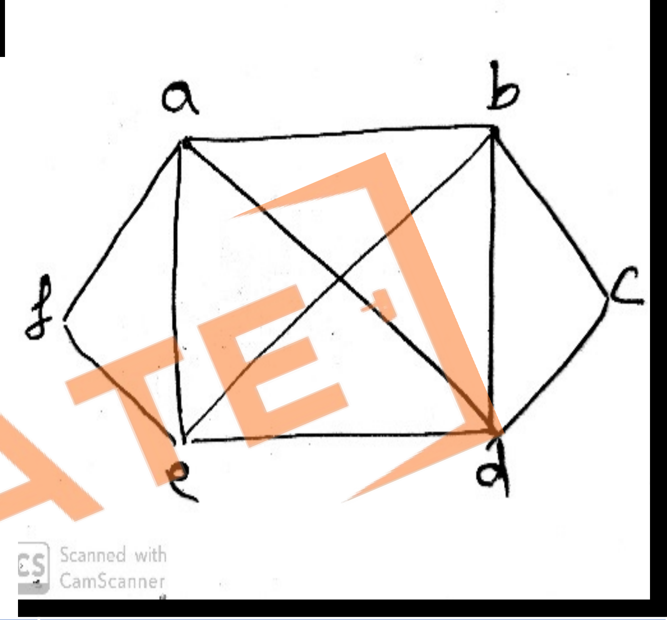
2. Finding weather a graph is Hamiltonian or not is a NPC problem.











Euler Graph				
Hamiltonian Graph				

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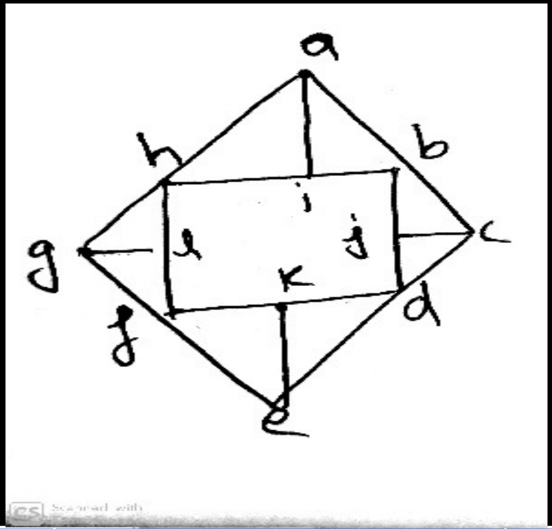

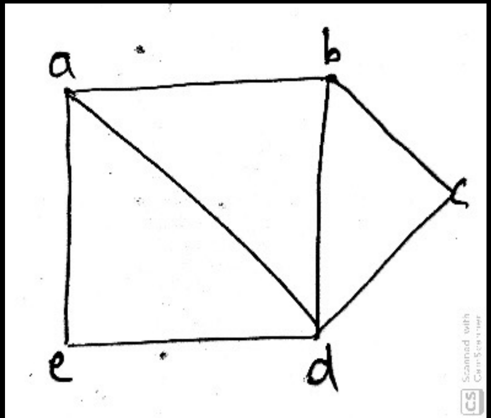

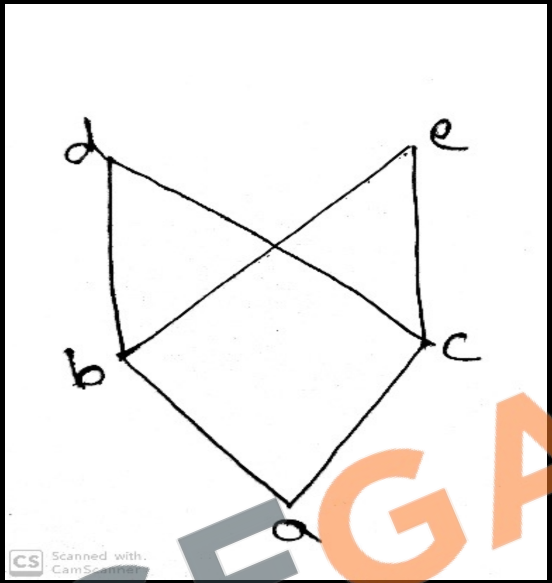

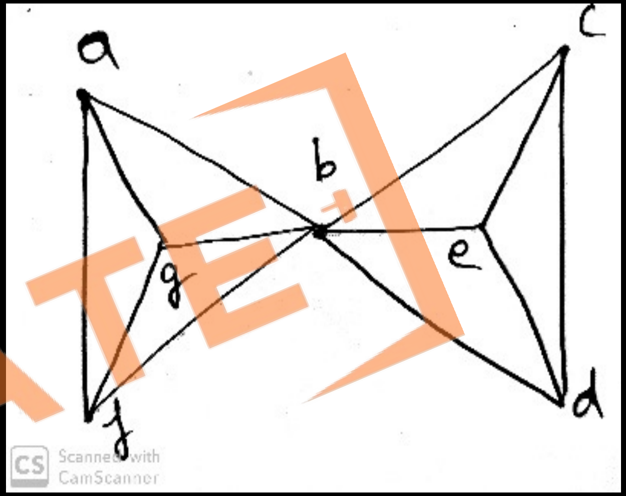

Euler Graph				
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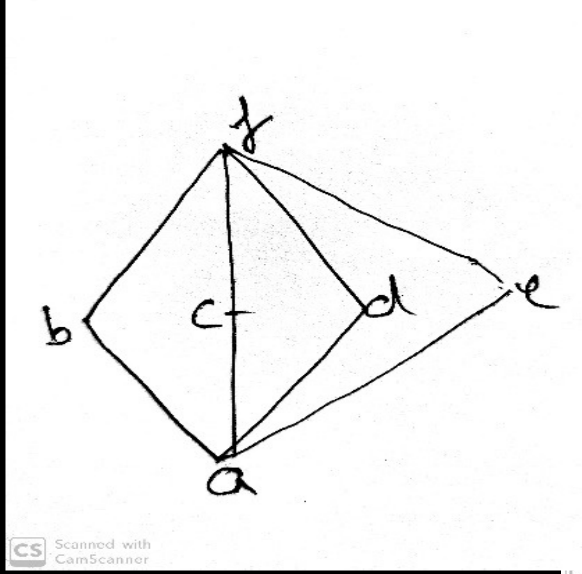

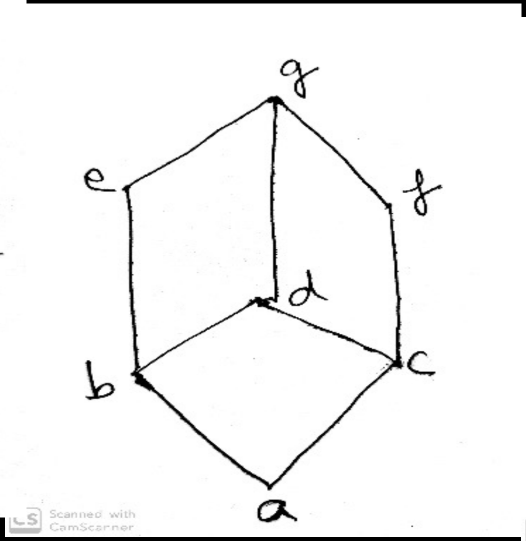

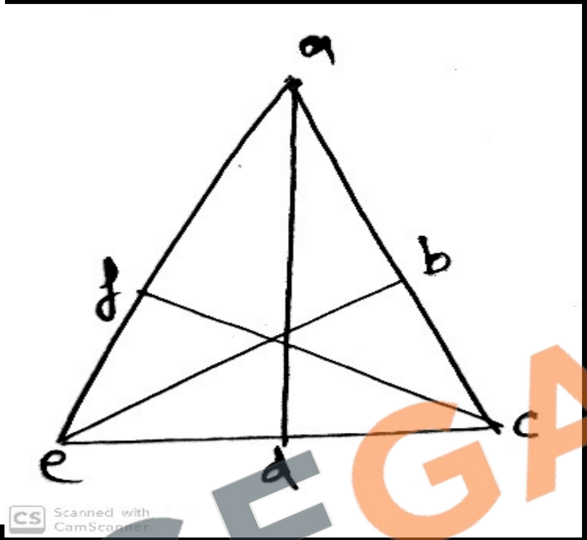

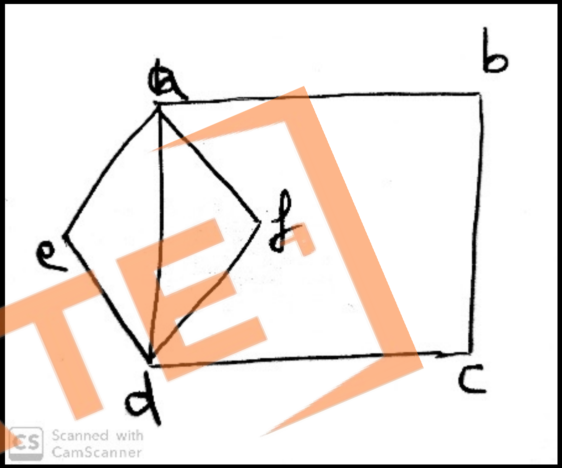

Euler Graph				
Hamiltonian Graph				

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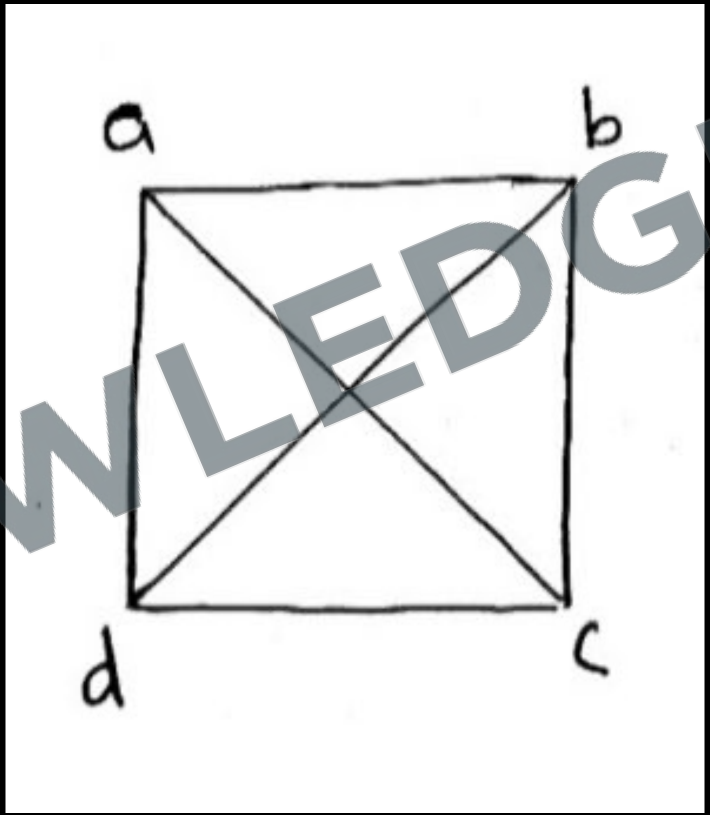
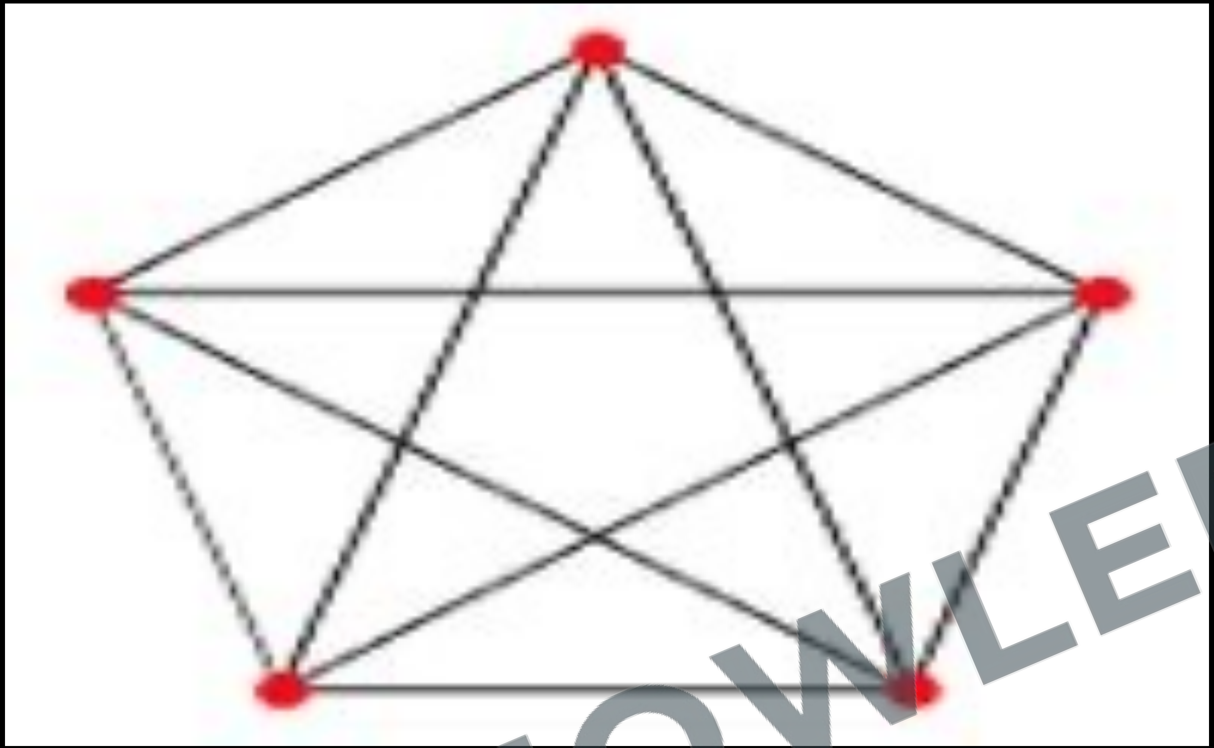
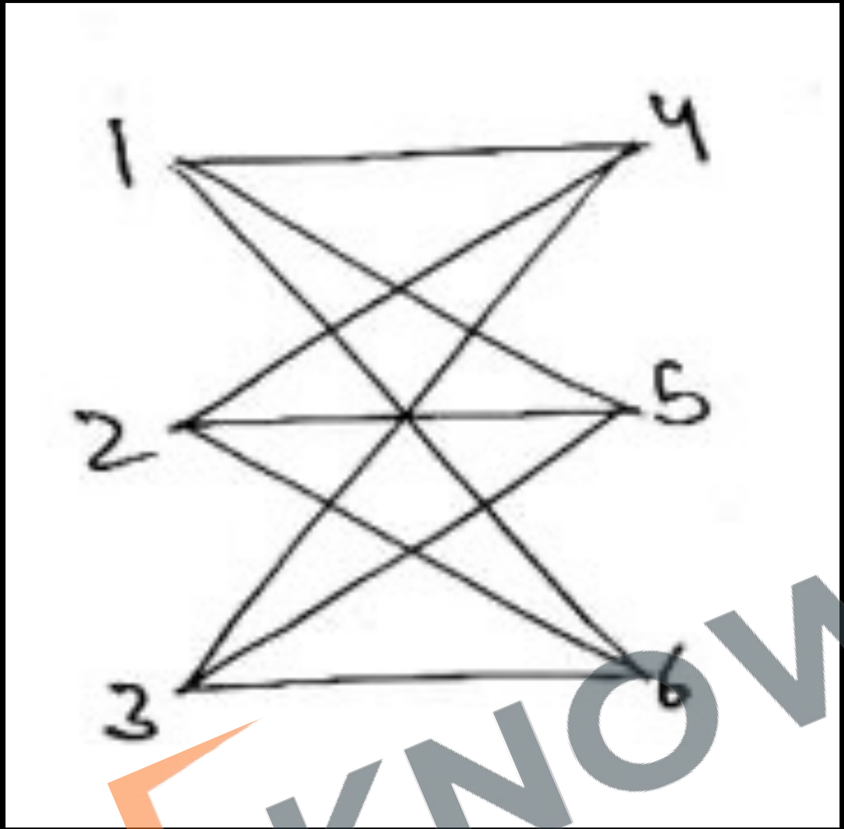
Planer Graph

Planer Graph: - A graph is called a planer graph if it can be drawn on a plan in such a way that no edges cross each other, otherwise it is called non-planer. Application: civil engineering, circuit designing



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Simplest Non-Planer Graphs

1. **Kuratowski's case I:** - K_5
2. **Kuratowski's case II:** - $K_{3,3}$
3. Both are simplest non-planer graph
4. Both are regular graph
5. If we delete either an edge or a vertex from any of the graph, they will become planer

- **Kazimierz Kuratowski** (2 February 1896 – 18 June 1980) was a Polish mathematician and logician.

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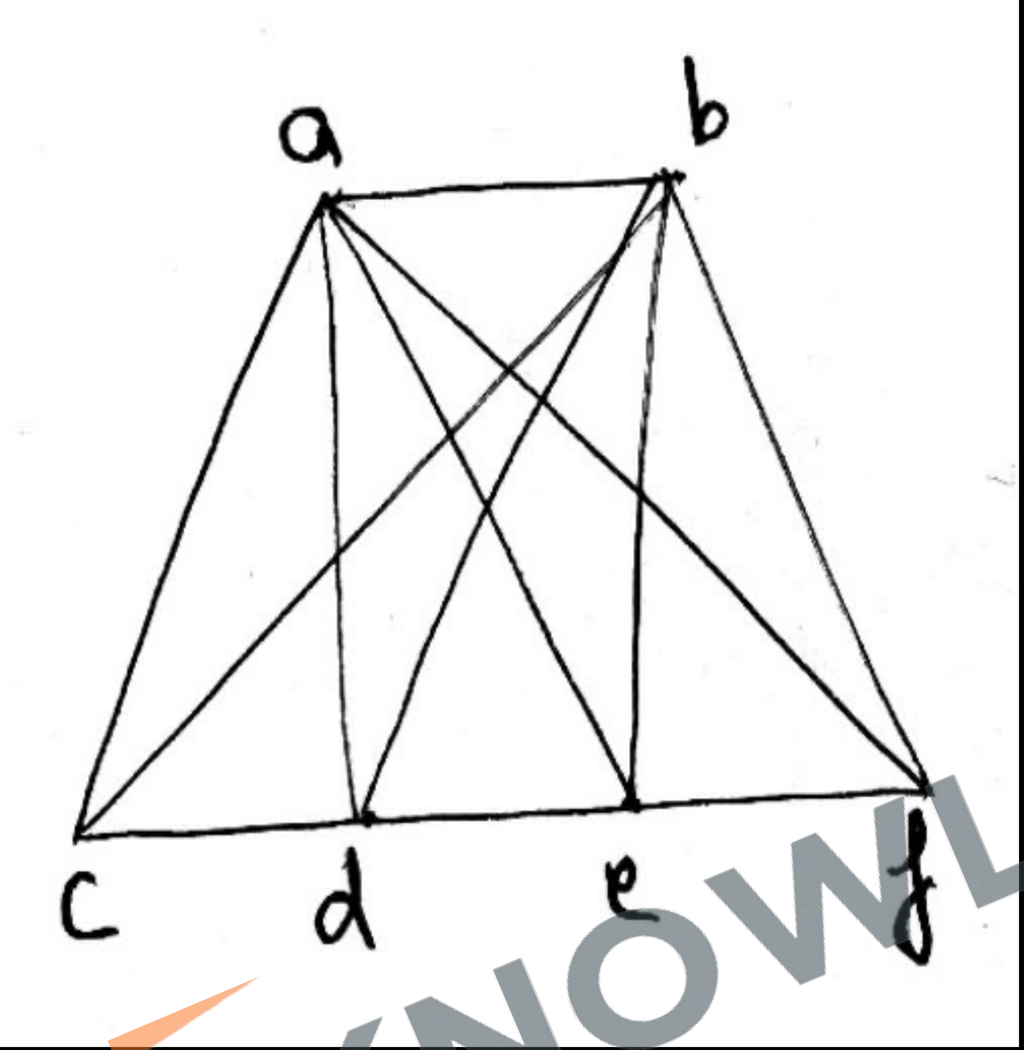
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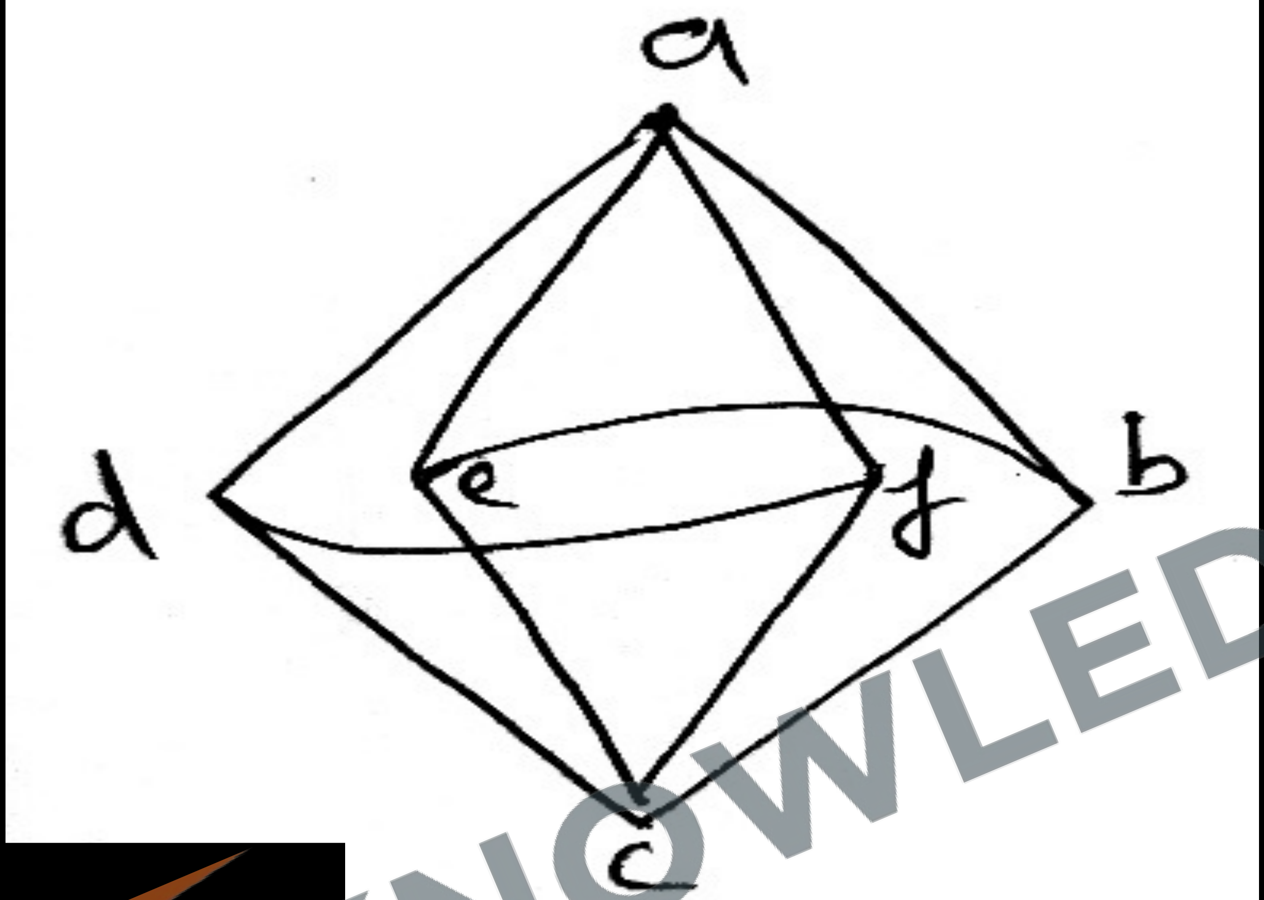
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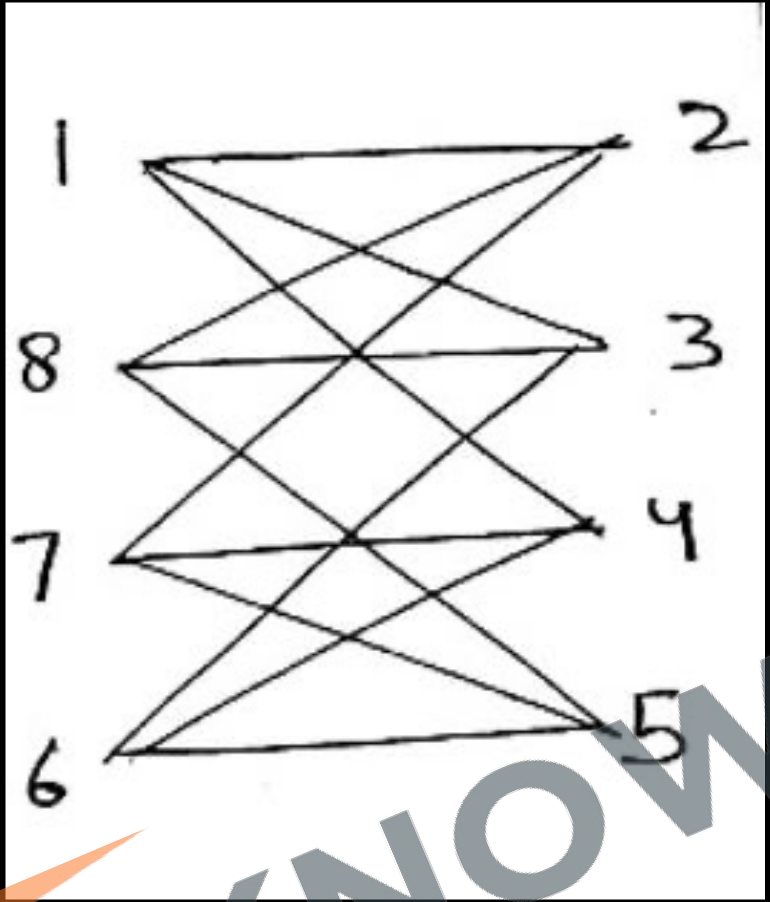
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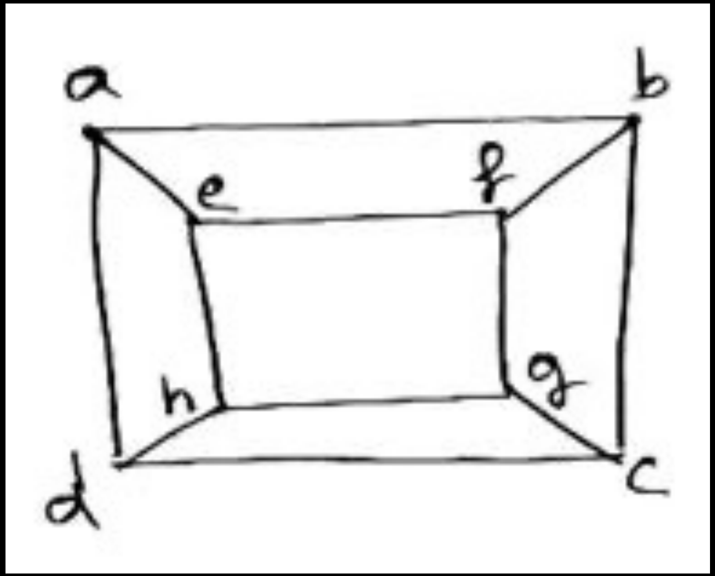

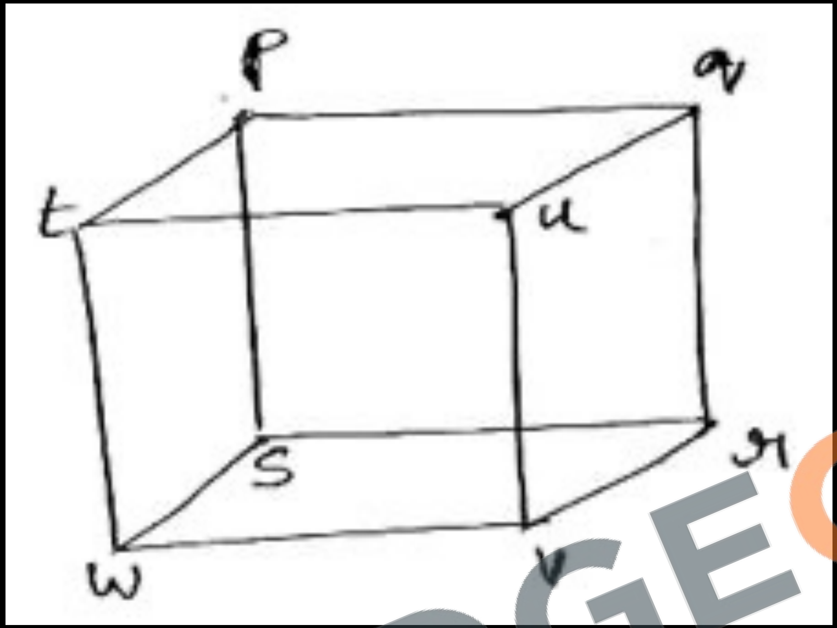

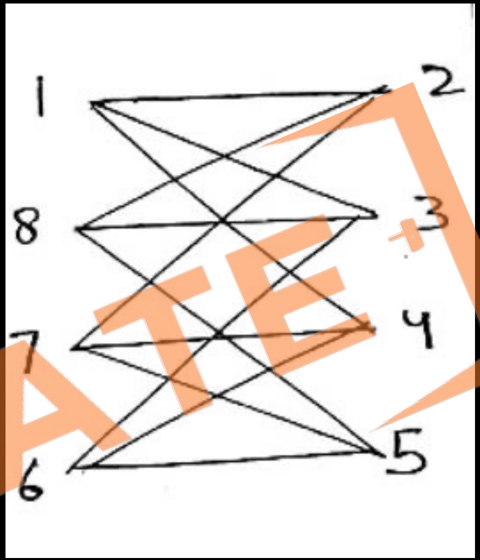

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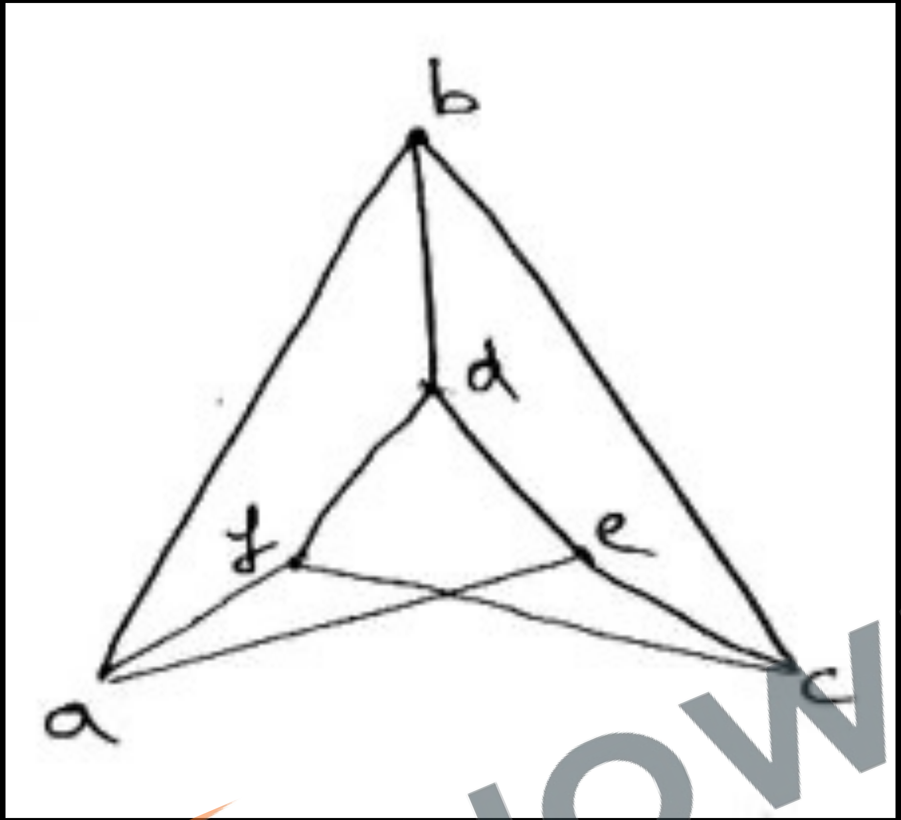
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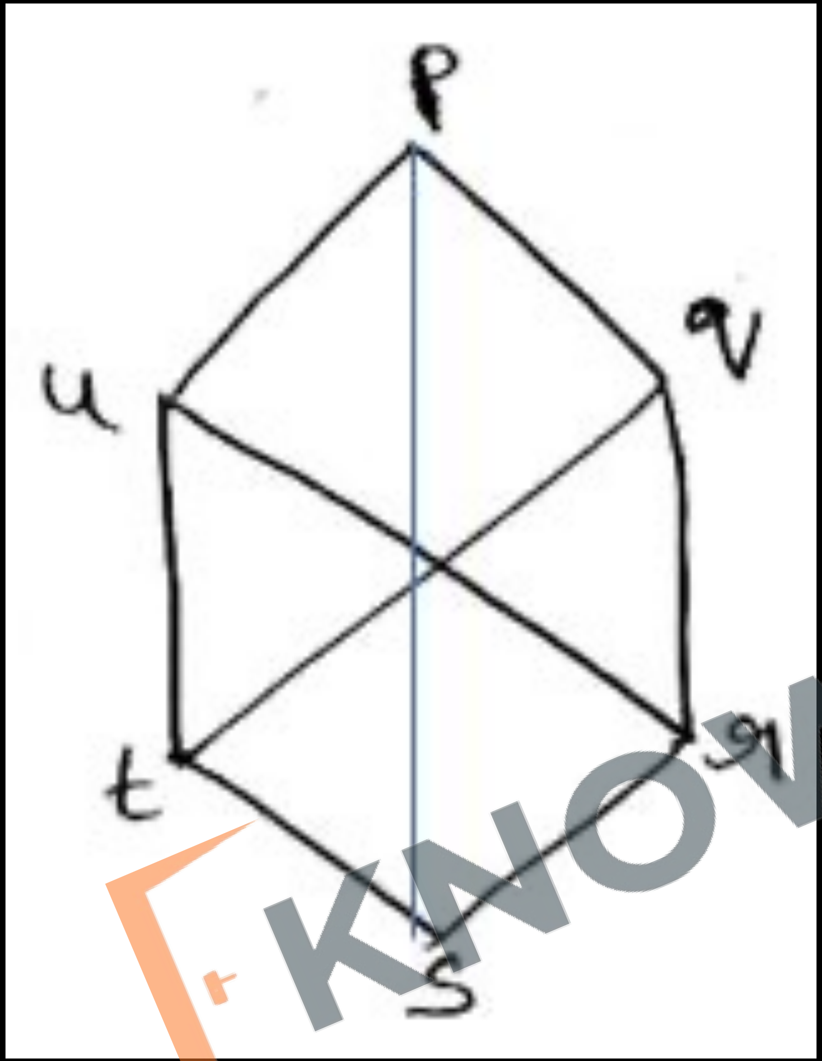
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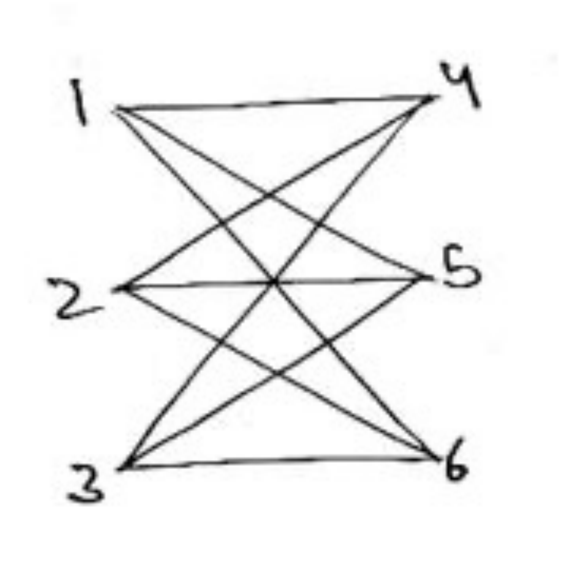

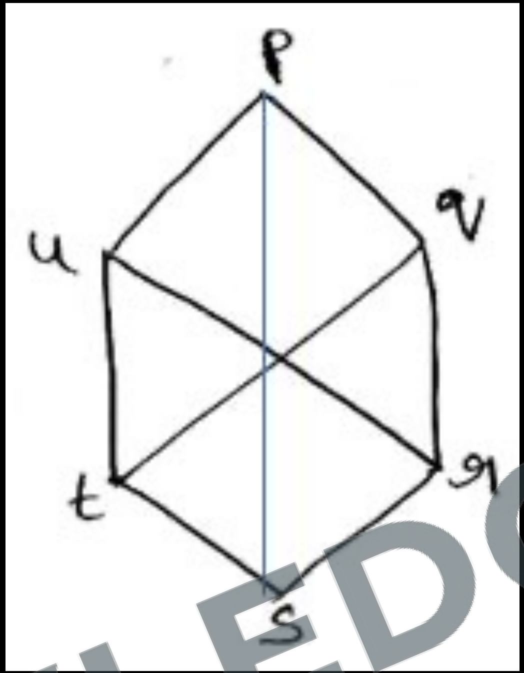

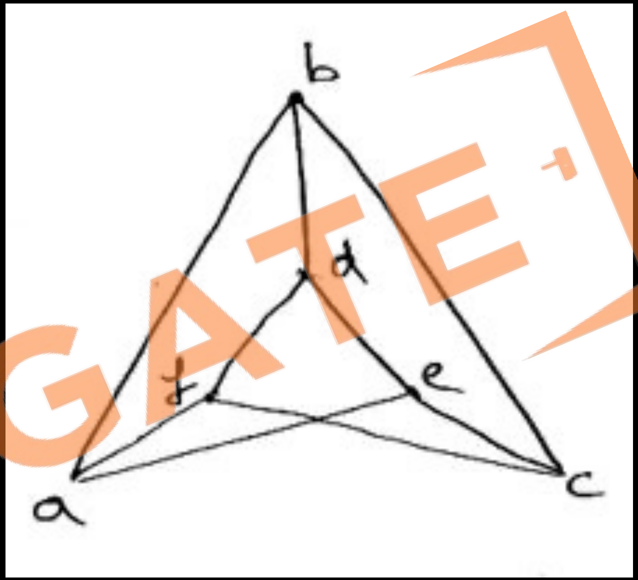

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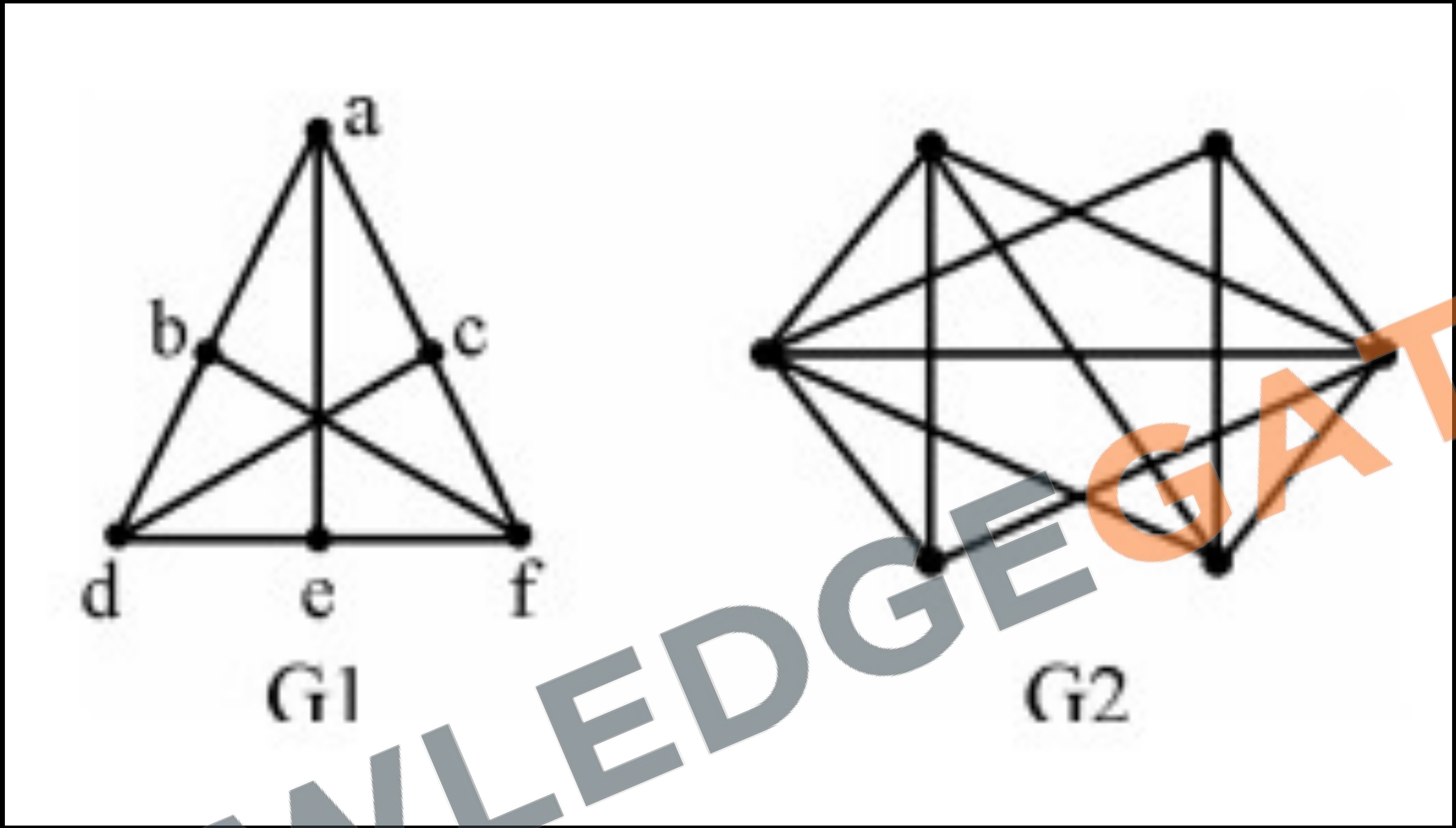

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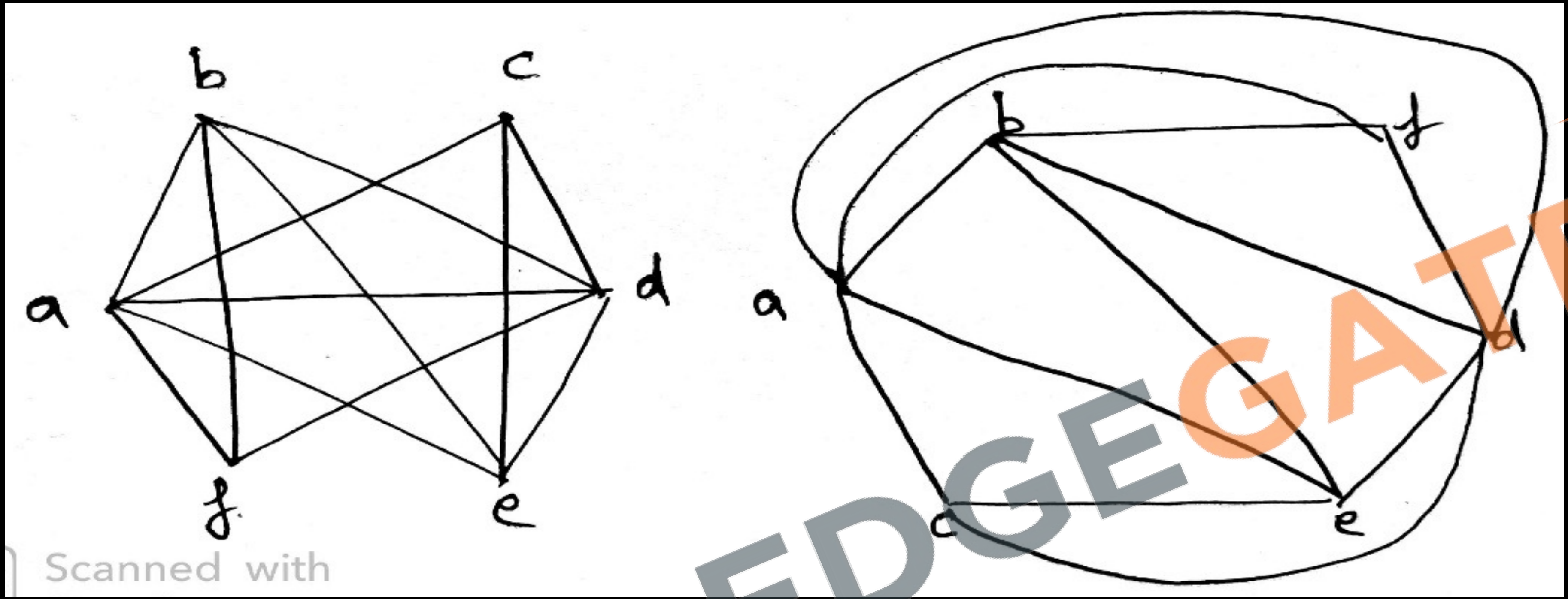
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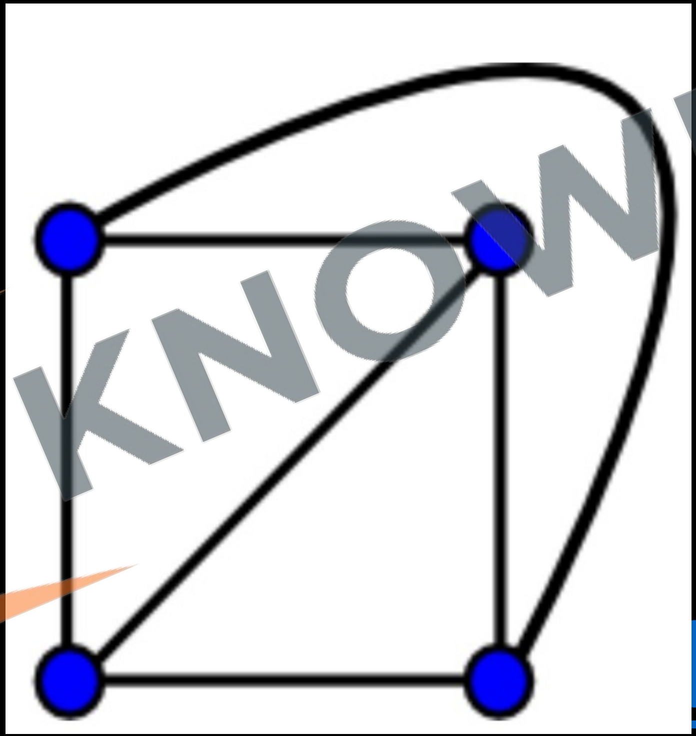
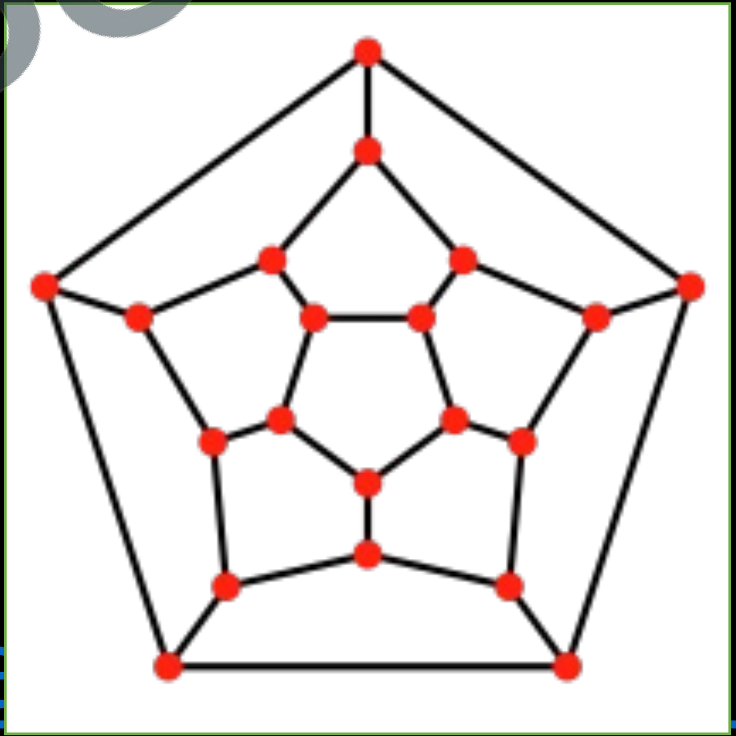
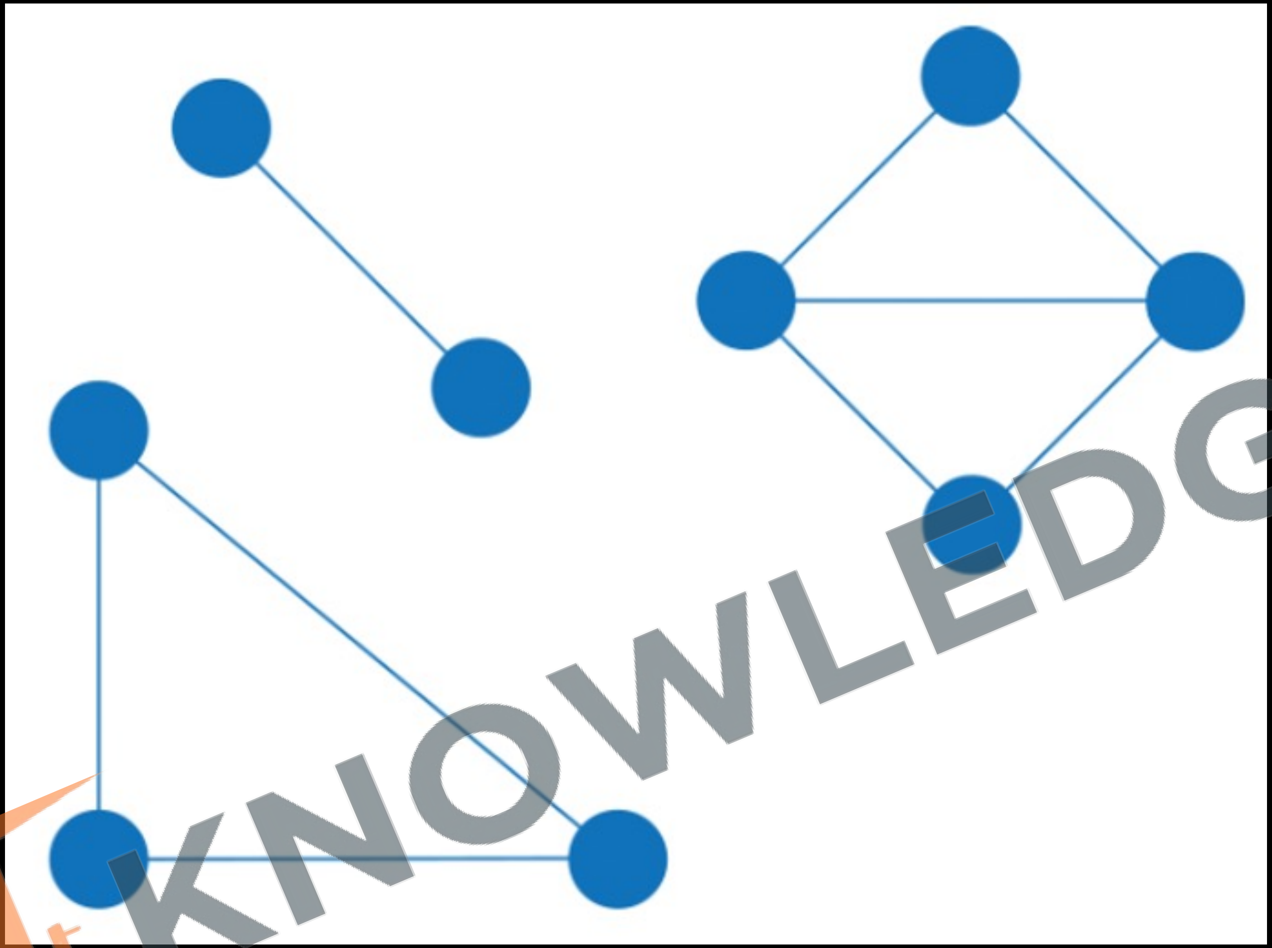
Euler's formula

- A planer graph divides the plane into number or regions (faces, planer embedding), which are further divided into bounded(internal) and unbounded region(external).
- **Euler's formula** states that if a finite, connected, planar graph with *v* is the number of vertices, *e* is the number of edges and *r* is the number of faces (regions bounded by edges, including the outer, infinitely large region), then
- $r = e - v + 2$
- Euler's formula can be proved by mathematical induction





- Euler's formula (Disconnected graph): $V - e + r - k = 1$



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Q In an undirected connected planar graph G, there are eight vertices and five faces. The number of edges in G is __________.

(a) 10

(b) 11

(c) 12

(d) 6

Q Let G be a simple connected planar graph with 13 vertices and 19 edges. Then, the number of faces in the planar embedding of the graph is

(A) 6 **(B)** 8 **(C)** 9 **(D)** 13

Other formula derived from Euler's formula

- Connected planar graphs with more than one edge obey the inequality $2e>=3r$, because each face has at least three face-edge incidences and each edge contribute exactly two incidences.

- Degree of the region is number of edges covering the region. Sum of degree of regions = $2|E|$

Using r = e – v + 2 and 3r<=2e

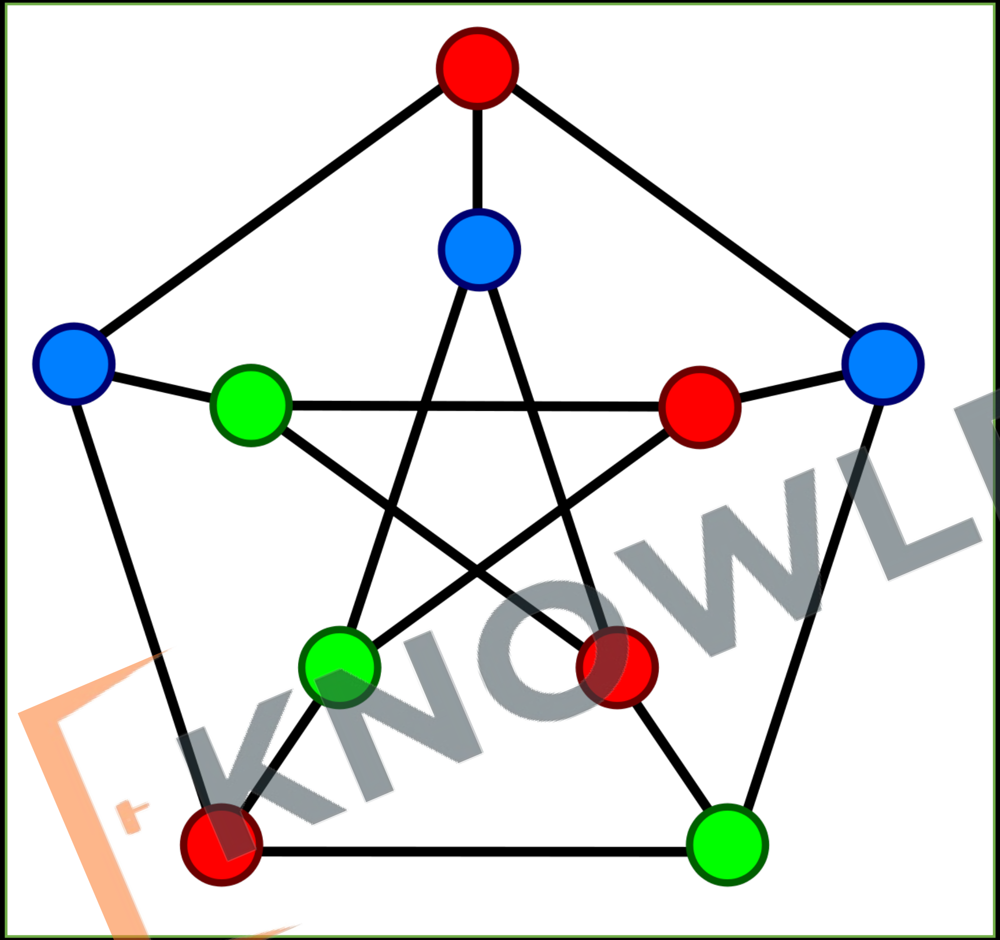
eliminating r we get, $e \leq 3v - 6$

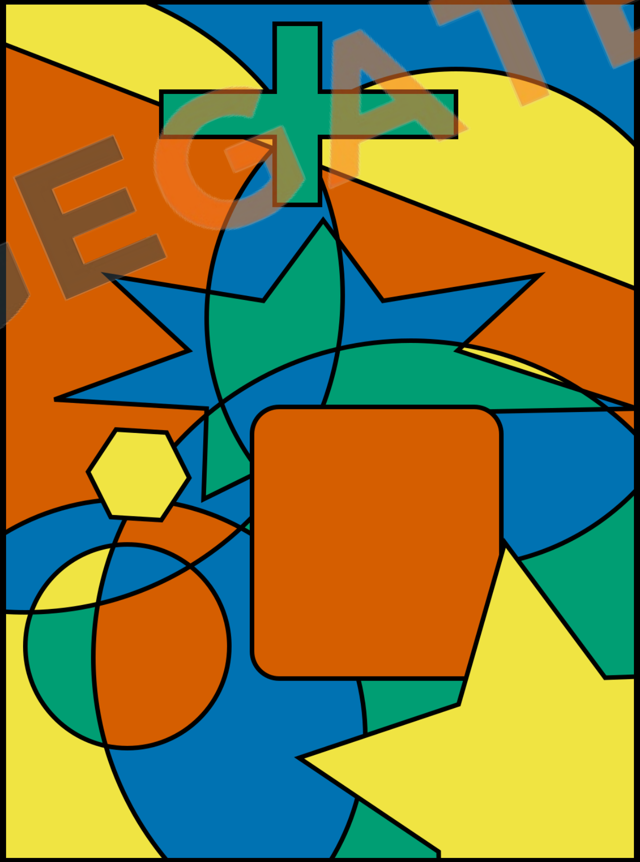
Using r = e – v + 2 and 3r<=2e

Eliminating e we get, $r \leq 2v - 4$

Graph Coloring

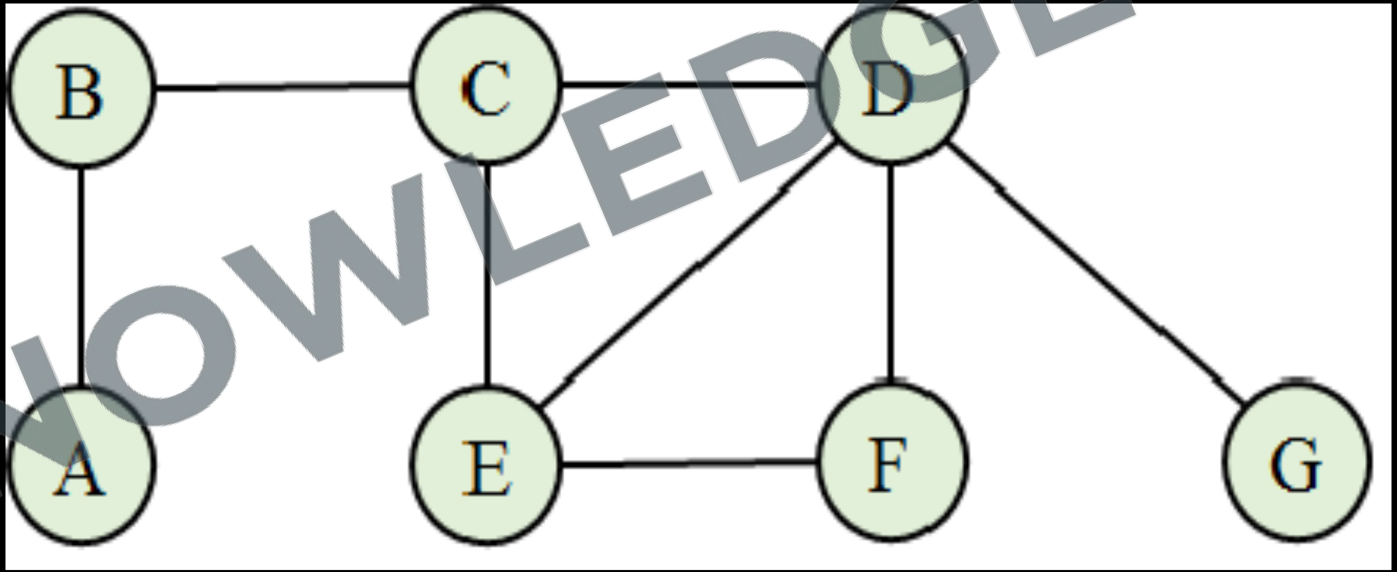
- Graph coloring can be of two types vertex coloring and region coloring.



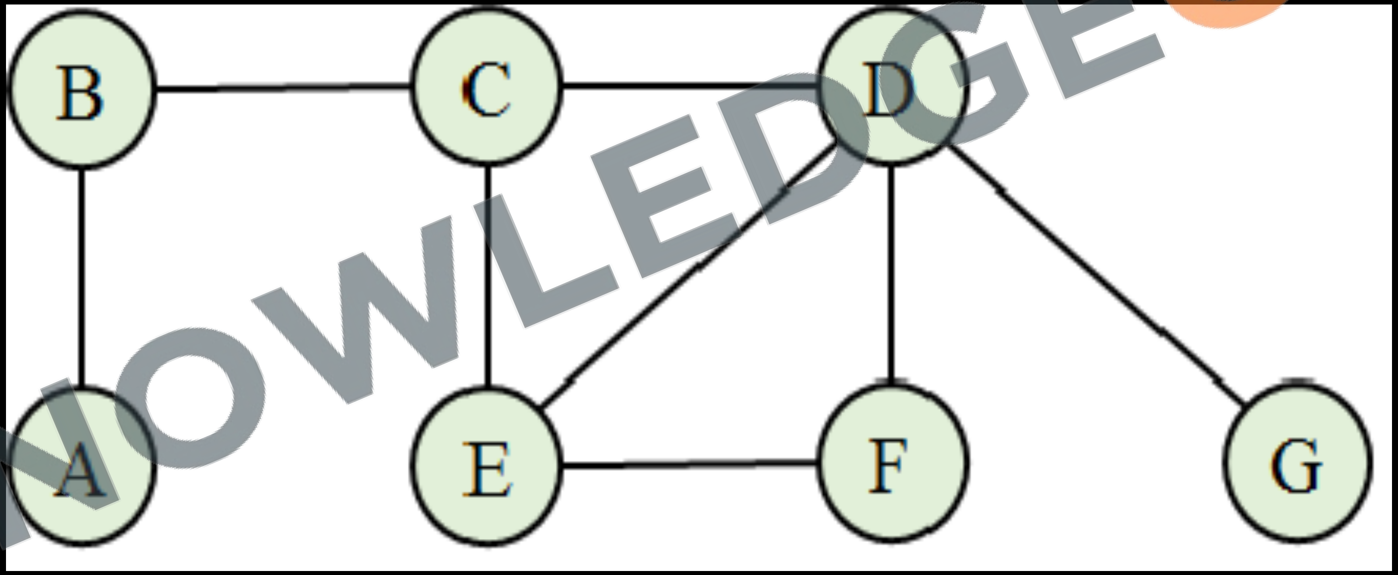


- Associating a color with each vertex of the graph is called vertex coloring.

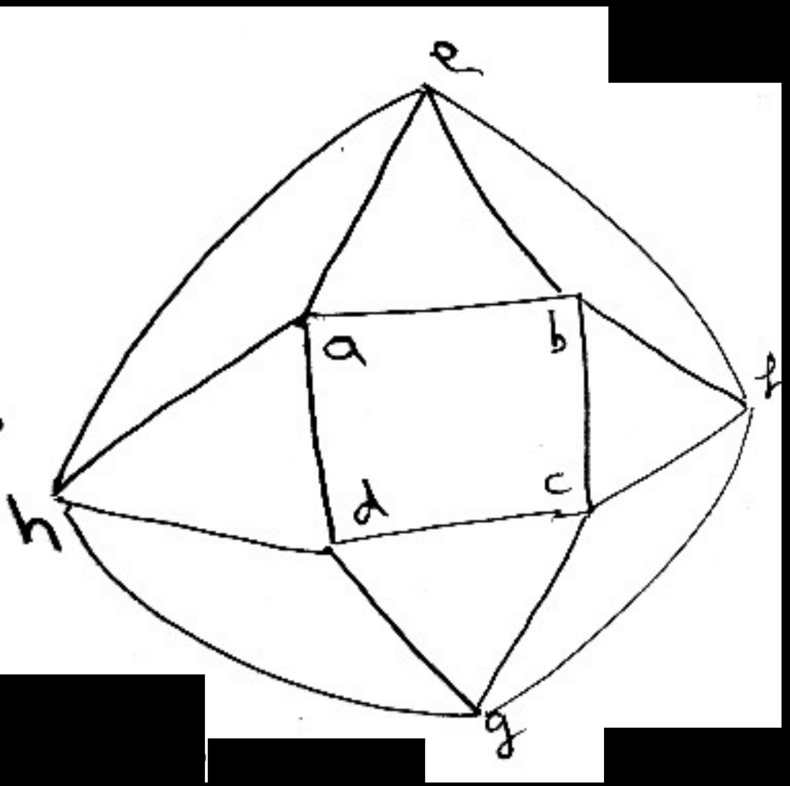
- **Proper Vertex coloring:** - Associating all the vertex of a graph with colors such that no two adjacent vertices have the same color is called proper vertex coloring.



- **Chromatic number of the graph:** - Minimum number of colors required to do a proper vertex coloring is called the chromatic number of the graph, denoted by $\chi(G)$. the graph is called K-chromatic or K-colorable.



- Cost of finding chromatic number is an NPC problem and there exists no polynomial algorithm to do that. There exists some greedy approach which try to solve it in P time, but they do not guarantee optimal solution.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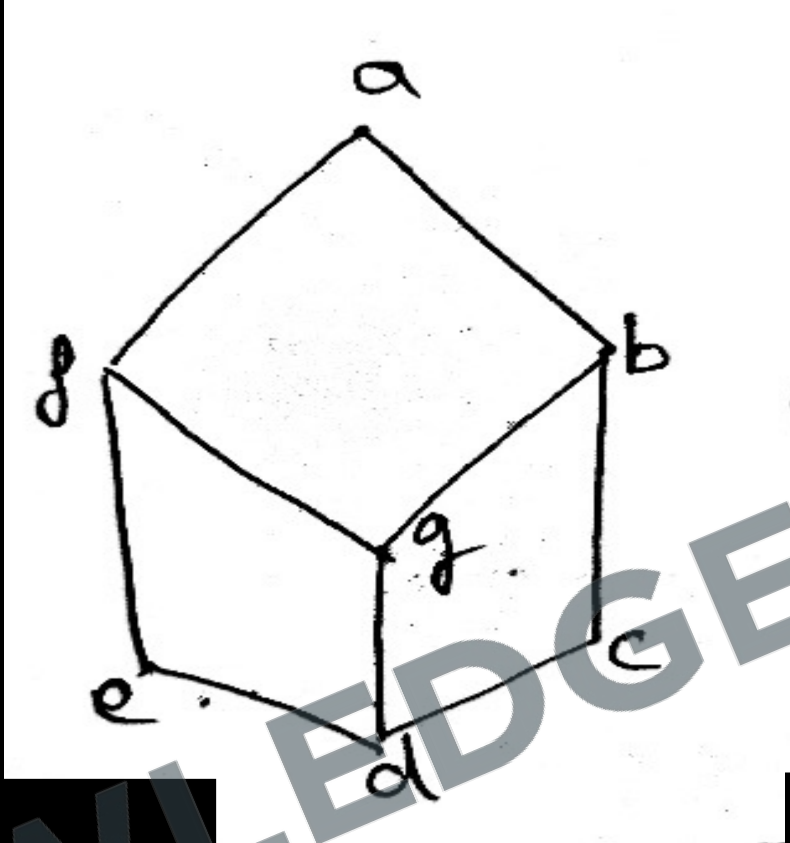
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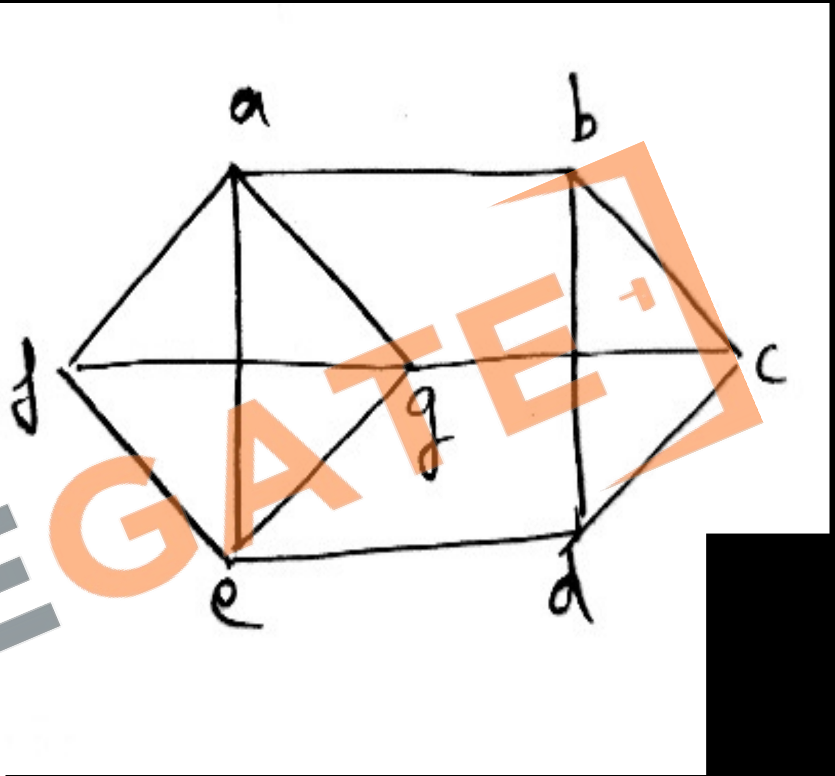
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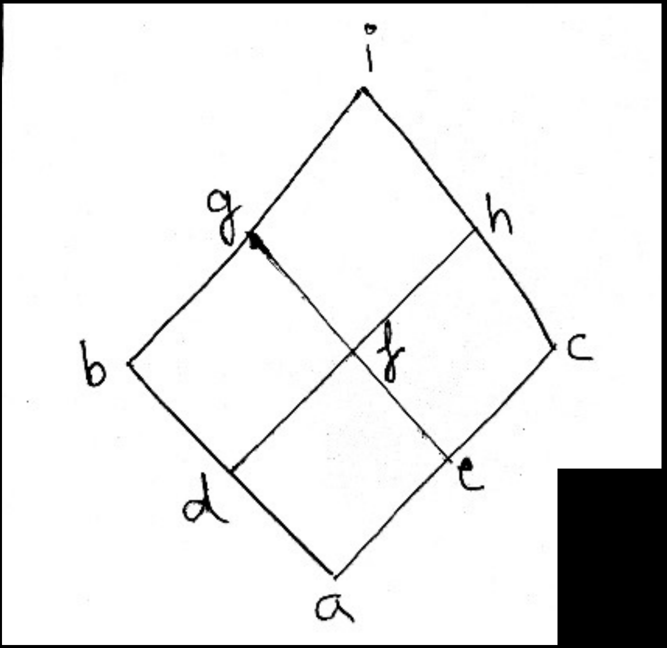
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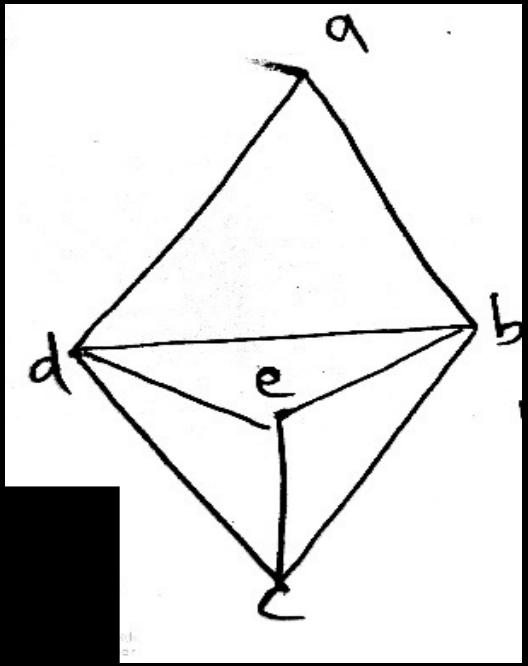
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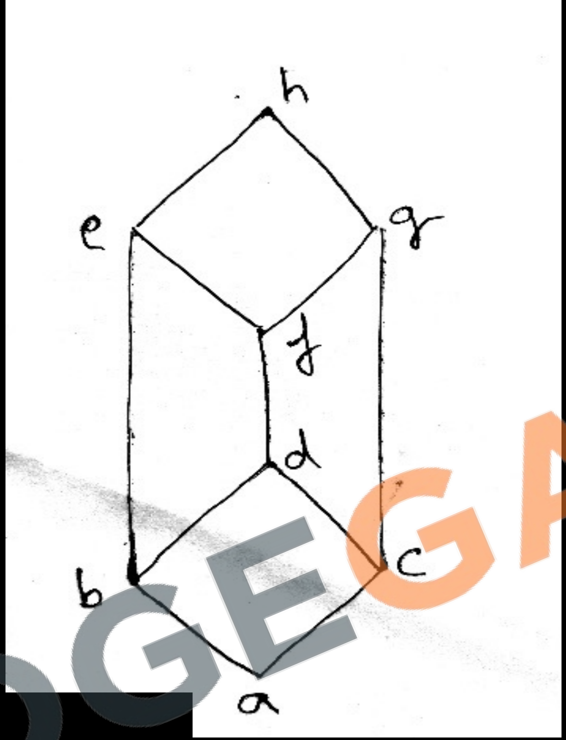
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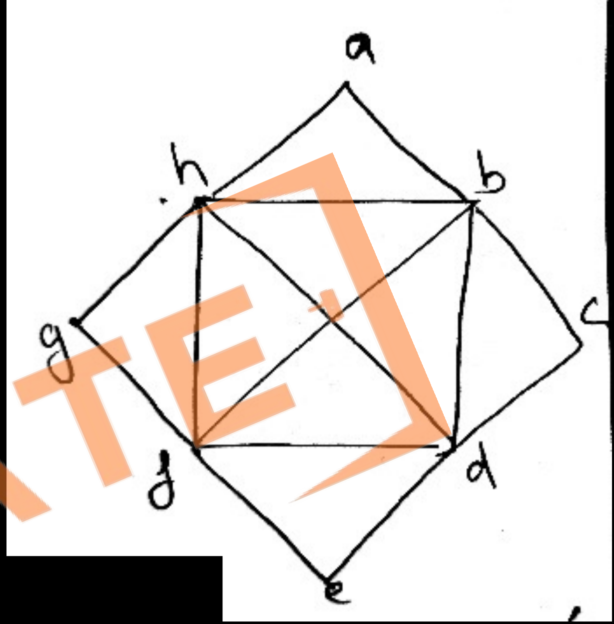
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Q The minimum number of colors that is sufficient to vertex color any planar graph is _______________

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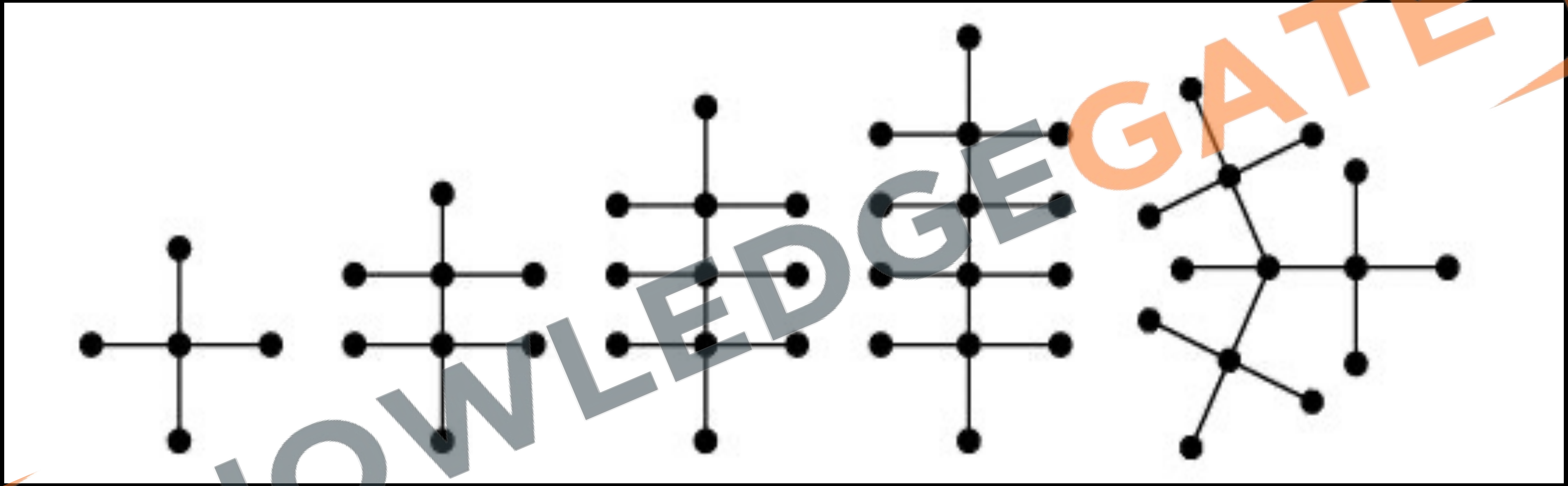
- Trivial graph is 1-chromatic
- A graph with 1 or more edge is at least 2-chromatic
- A complete graph K_n is n-chromatic

- Tree is always 2-chromatic
- Bi-partite graph is 2-chromatic
- C_n is 2-chromatic if n is even, C_n is 3-chromatic if n is odd

- 5-color theorem-any planer graph is at most 5-chromatic
- 4-colour theorem/hypothesis- any planer graph is 4-chromatic
- If Δ(G) is the maximum degree of any vertex in a graph then, $\chi(G) <= 1 + \Delta(G)$

Tree

- A tree is a connected graph without any circuit.



- There is one and only one path between every pair of vertices in a tree

- If in a graph G, there is one and only one path between every pair of vertices then G is a tree

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- A tree with n vertices has n-1 edges

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- Any connected graph with n vertices and n-1 edges in a tree

- A graph is a tree if and only if it is minimally connected

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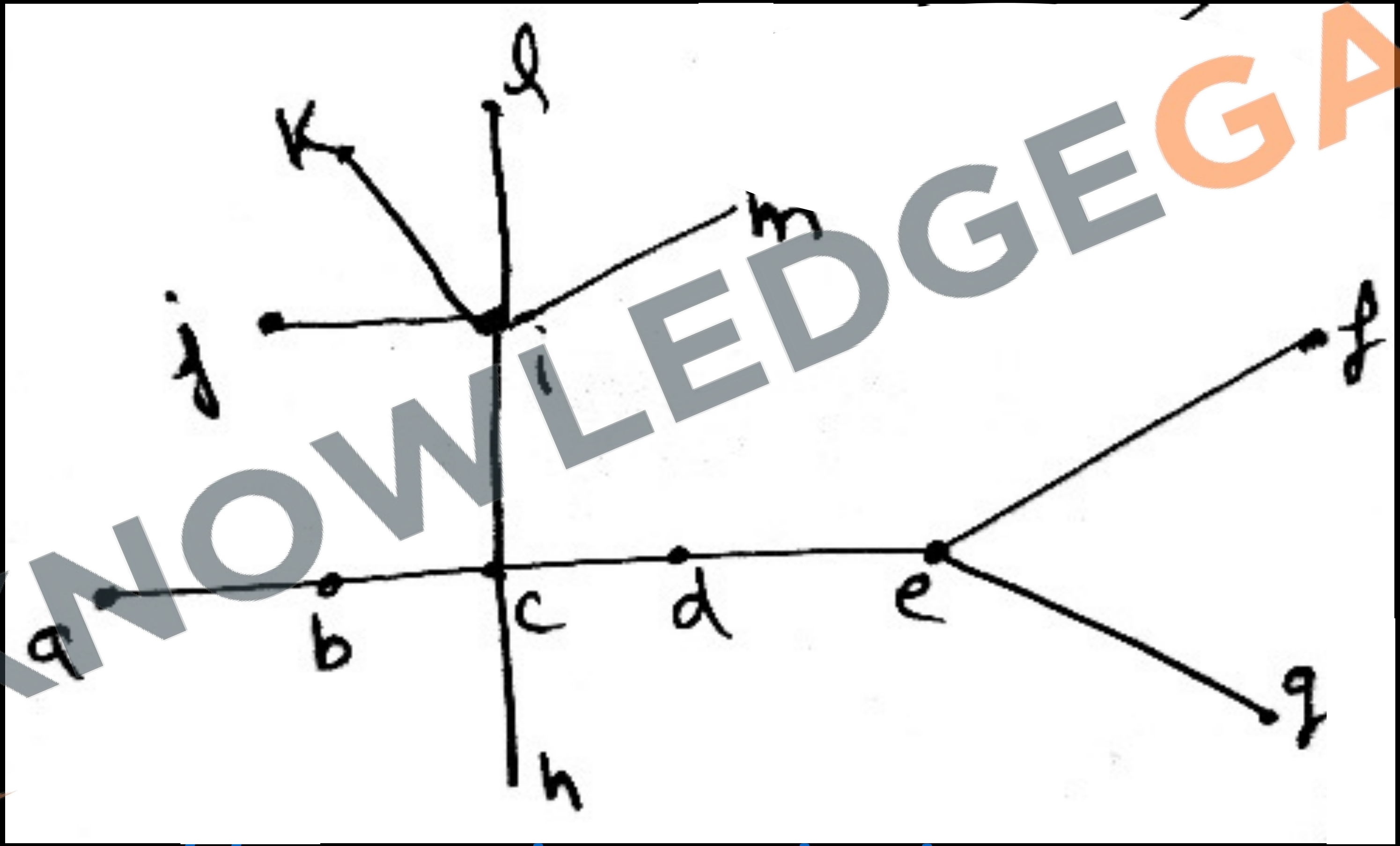
- A graph G with n vertices and n-1 edges and no circuit is connected

Eccentricity: - Eccentricity of a vertex is denoted by E(v) of a vertex v in a graph G, it is the distance from V to the vertex farthest from V in G. $E(v) = \max d(v, v_i)\ v_i \in G$



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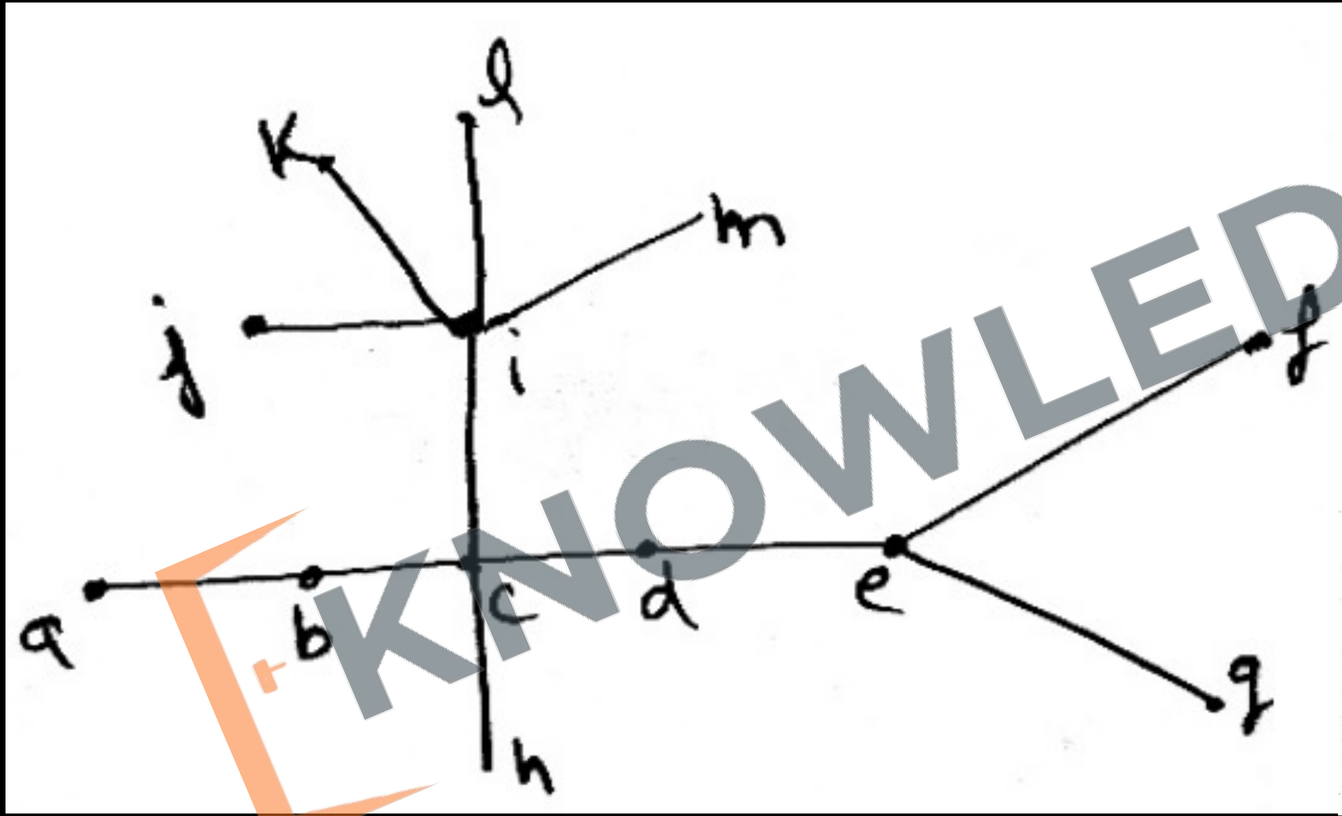
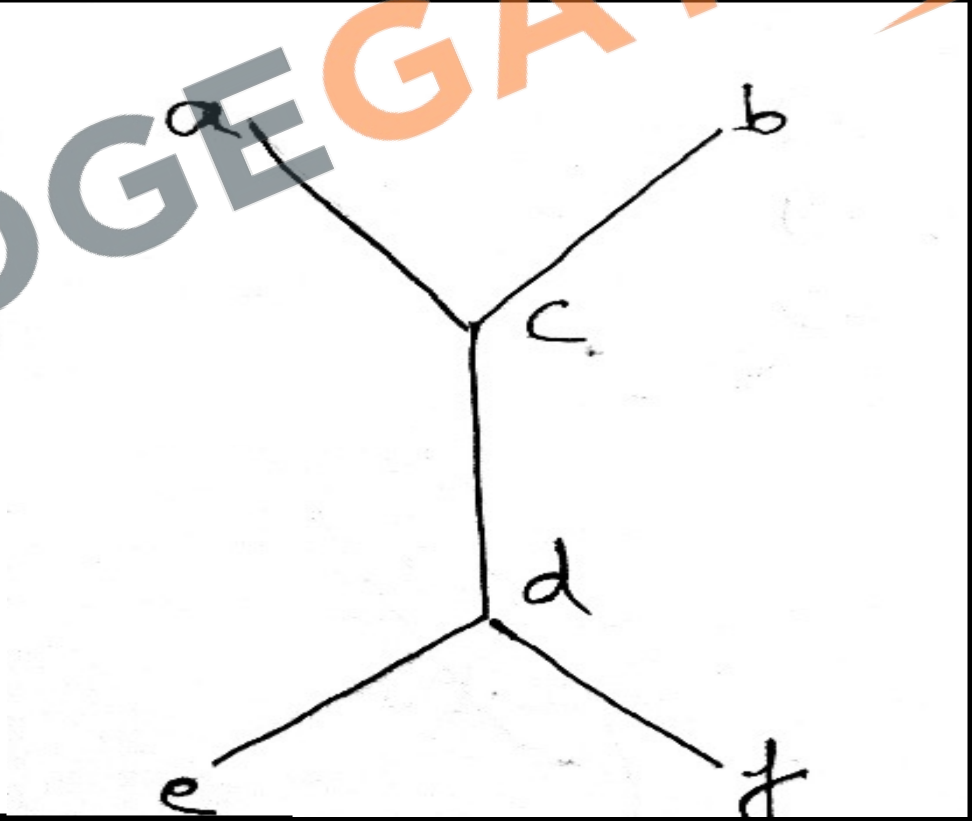
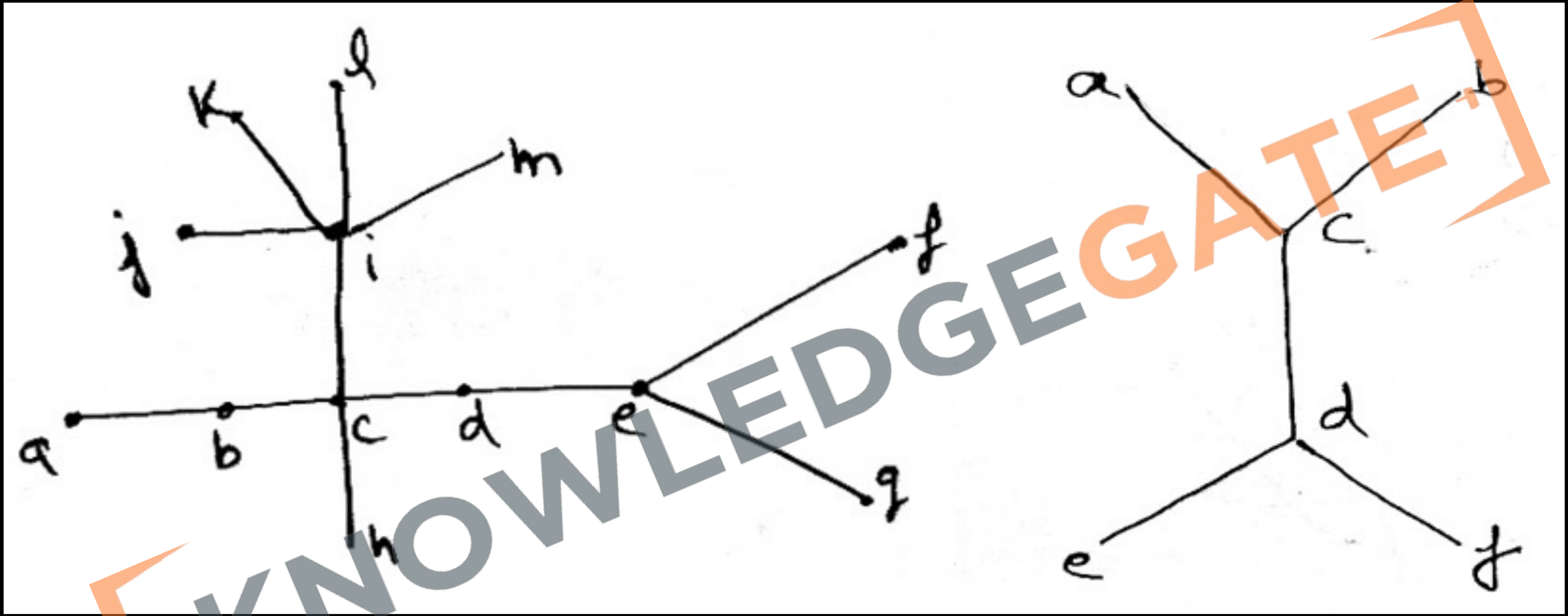
- A vertex with minimum eccentricity in a tree T is called center of T.
- Minimum eccentricity of any vertex in a tree T is called radius of tree. (eccentricity of center)
- Maximum eccentricity of any vertex in a tree T is called diameter of tree. (length of the longest path)





• Every tree has either one or two centers.



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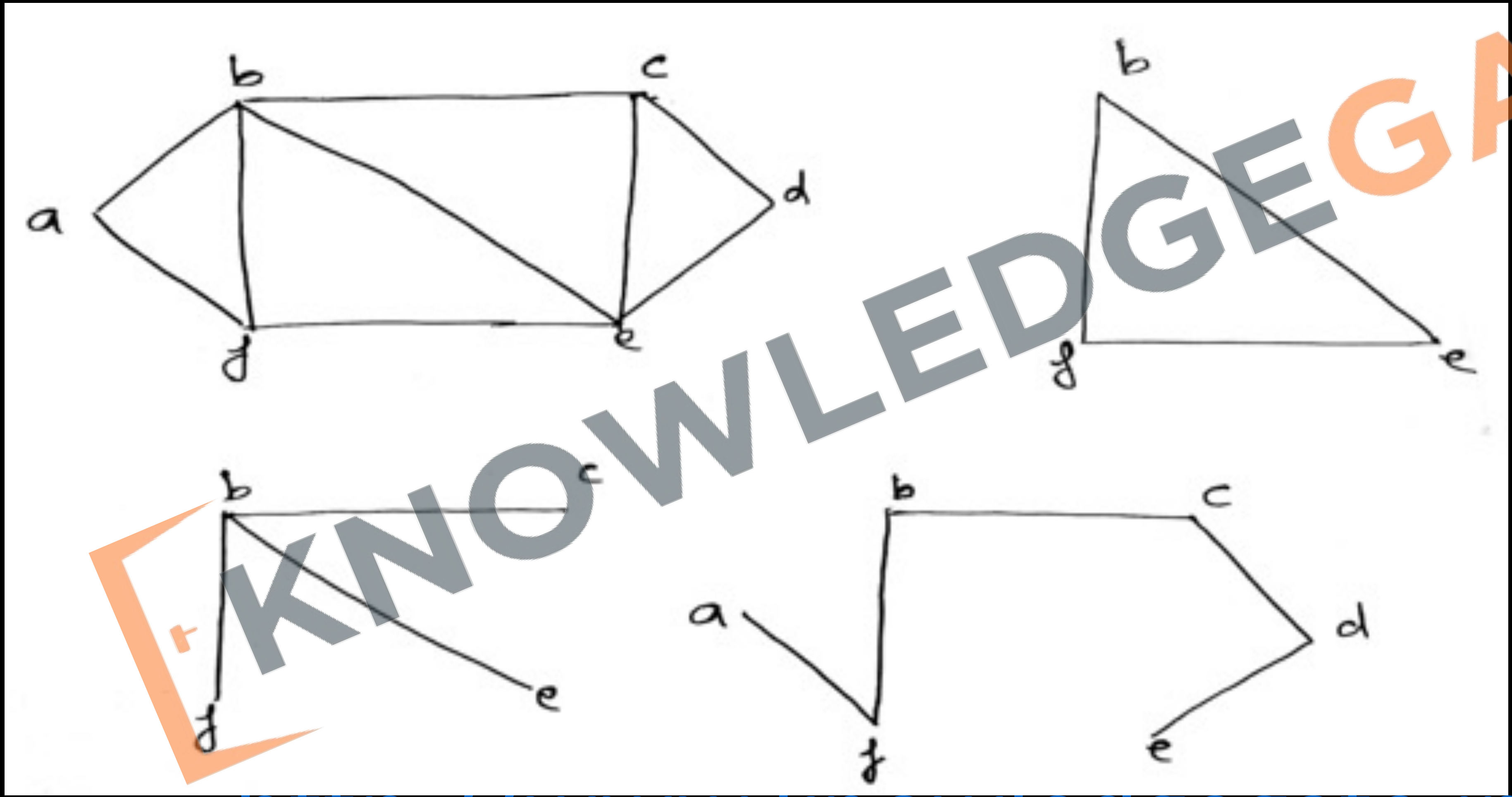
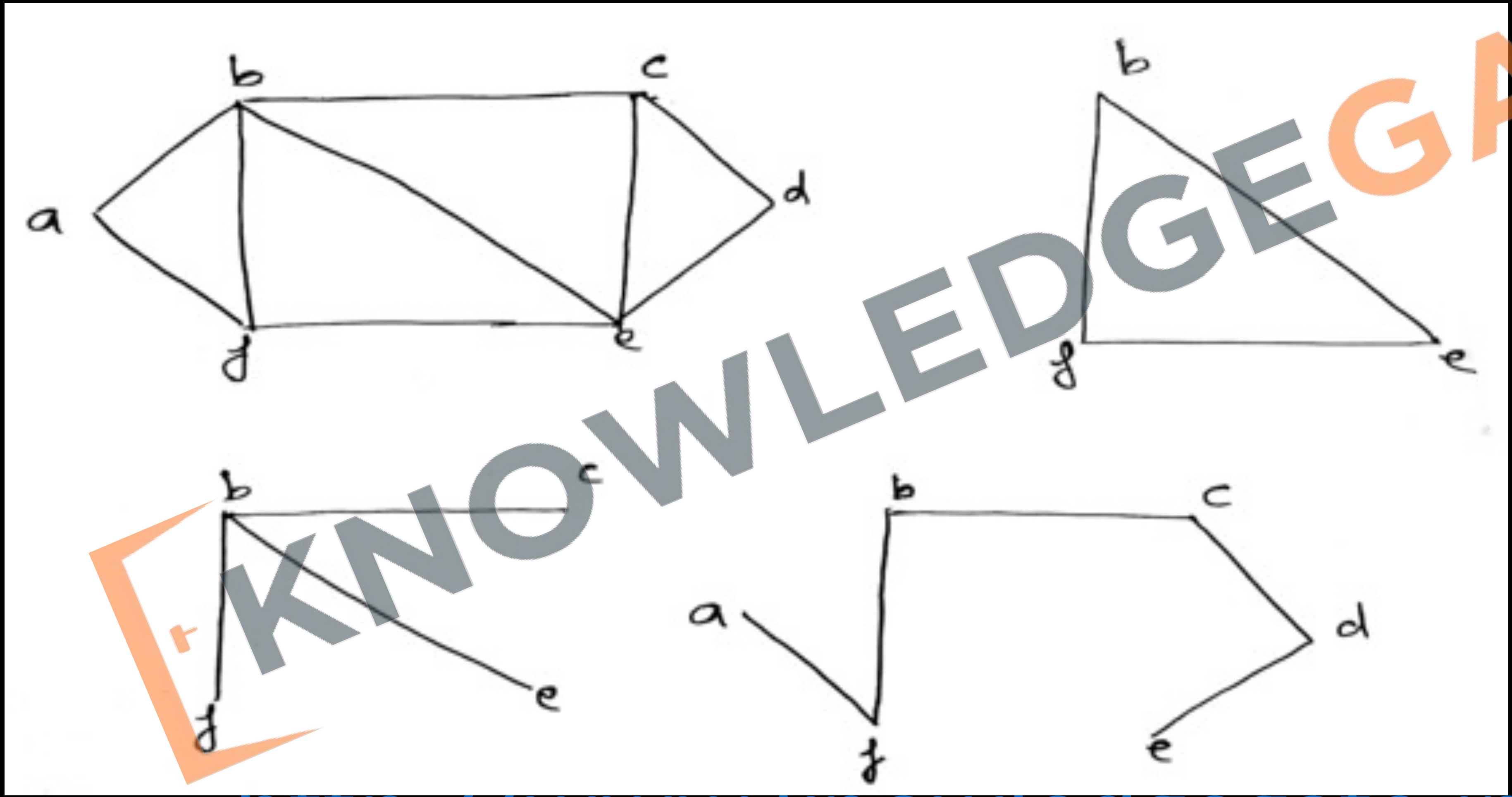
Q Let T be a tree with 10 vertices. The sum of the degrees of all the vertices in T is _____.

Spanning tree

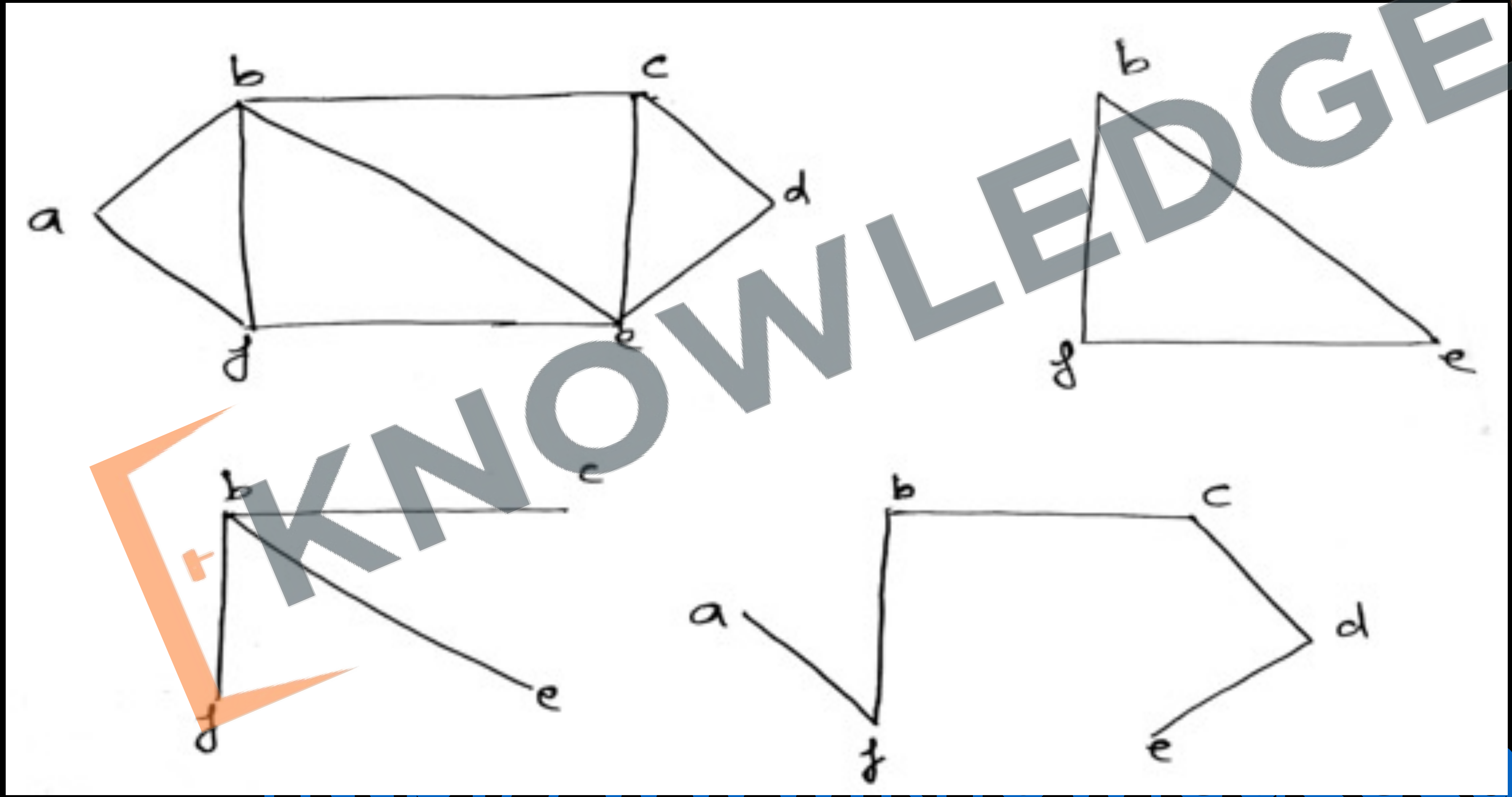
- A tree T is said to be spanning tree of a connected graph G, if T is a subgraph of G and T contains all vertices of G.



- An edge in a spanning tree T is called a branch of T
- An edge that is not in the given spanning tree T is called a chord.
- Branch and Chord are defined with respect to a given spanning tree.

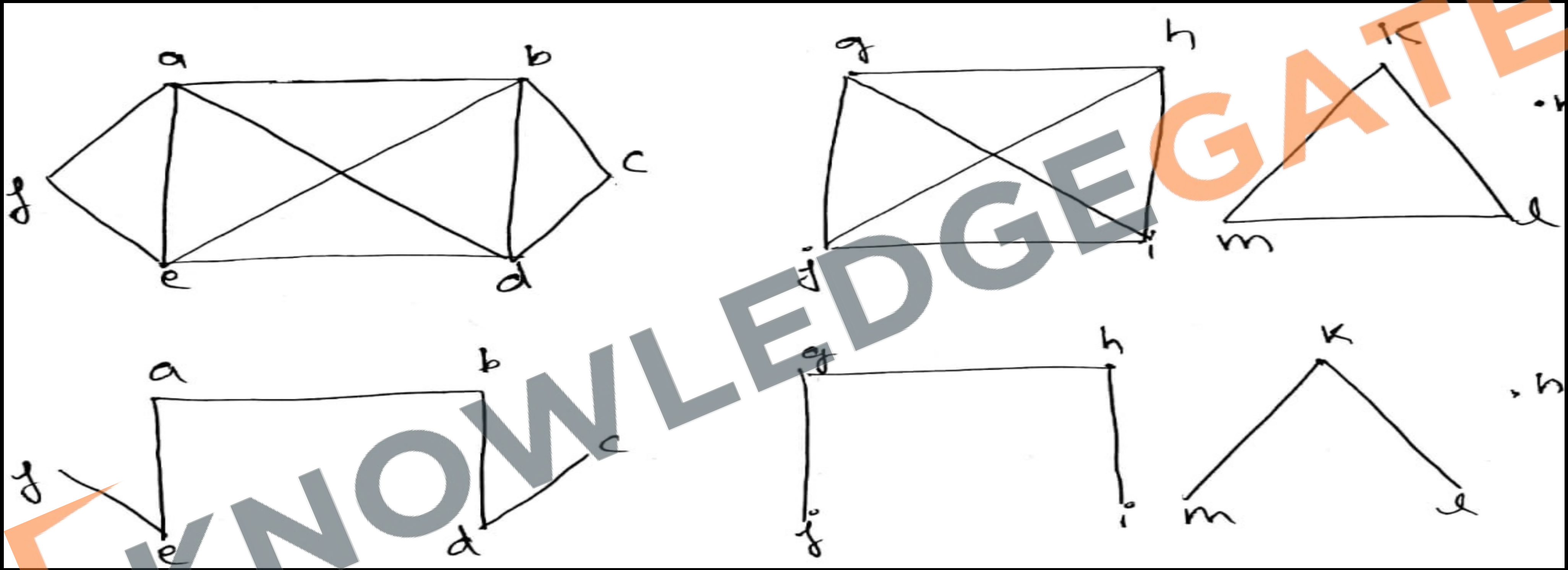


1. With respect to any of its spanning tree, a connected graph of n vertices and e edges has n-1 branches and e-n+1 chord
2. A connected graph G is a tree if and only if adding an edge between any two vertices in g creates exactly one cycle.
3. Rank(r) = n-1
4. Nullity(μ) = e – n + 1
5. Rank + nullity = number of edges in G

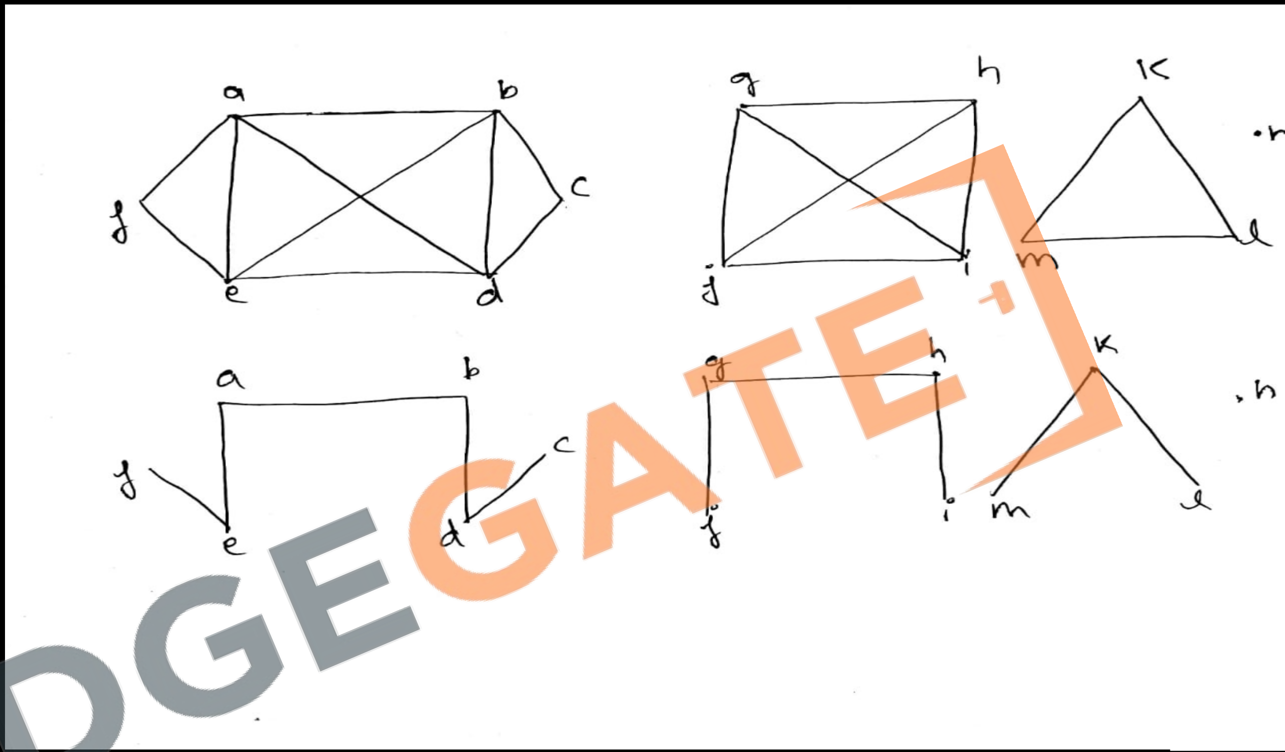


Spanning Forest: - if a graph is not connected, then there is no possibility of finding a spanning tree, but we can find a spanning forest. If a graph is not connected then we can find connected components, finding a spanning tree in each component we can find spanning forest.

A disconnected graph with K components has a spanning forest consisting of K spanning tree.



1. Rank(r) = n-k
2. Nullity(μ) = e – n + k
3. Rank + nullity = number of edges in G



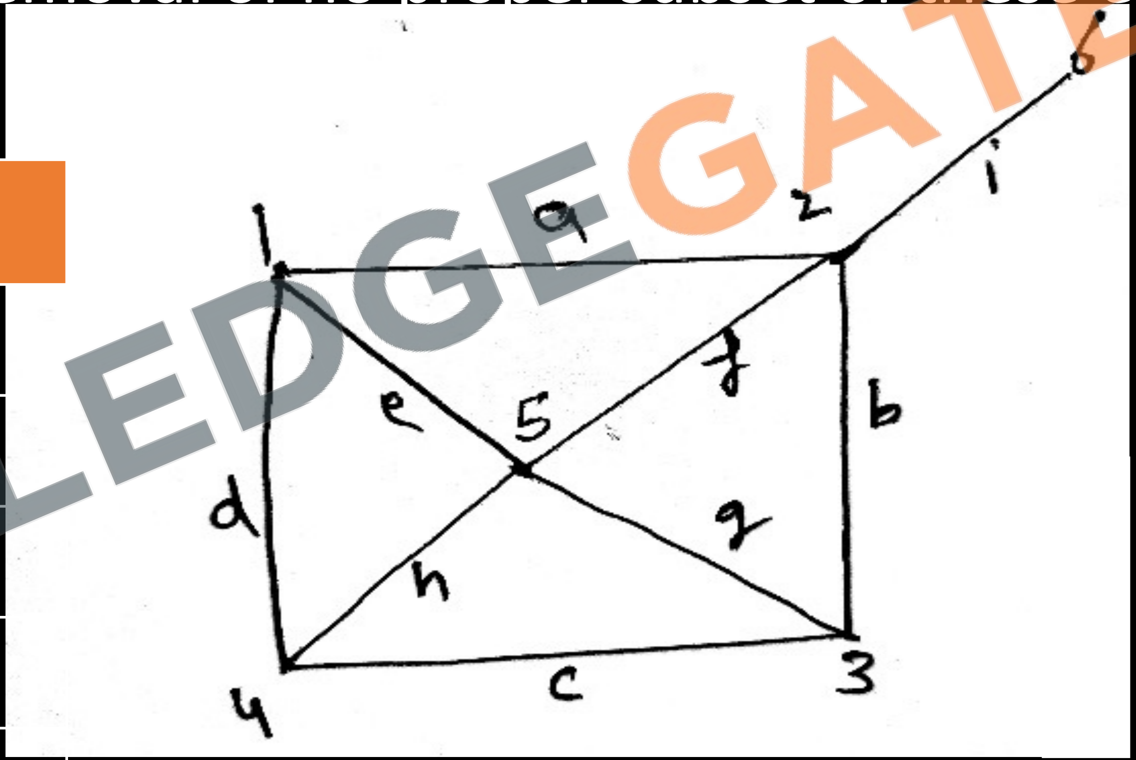
Fundamental circuit: - With respect to a spanning tree T in a connected graph G, adding any one chord to T will create exactly one circuit such a circuit formed by adding a chord to a spanning tree is called fundamental circuit.

Cut-Set (edge and vertex connectivity)

Cut-Set (Edges)

Cut Set: - In a connected graph G, a cut set is a set of edges whose removal from g leaves G disconnected, provided removal of no proper subset of these edges disconnects G.

Cut Set	Validity
{a, f, g}	
{a, e, h, c}	
{a, i}	
{e, h, f, g}	
{d, h, c, g}	
{d, e, f}	



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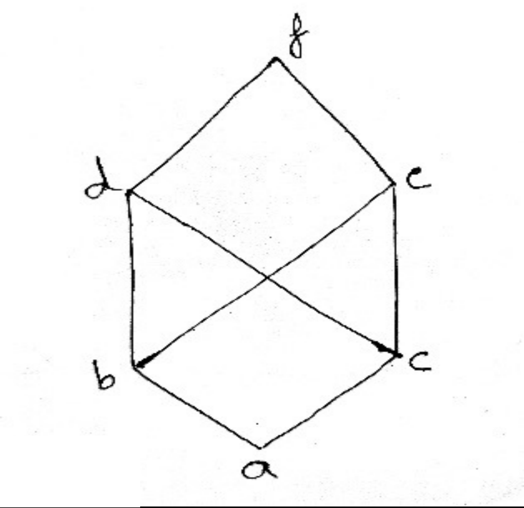
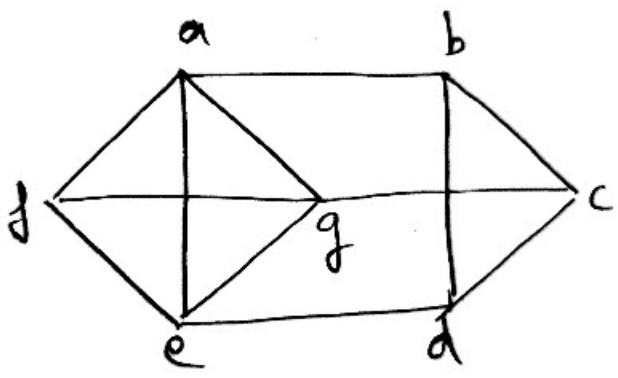
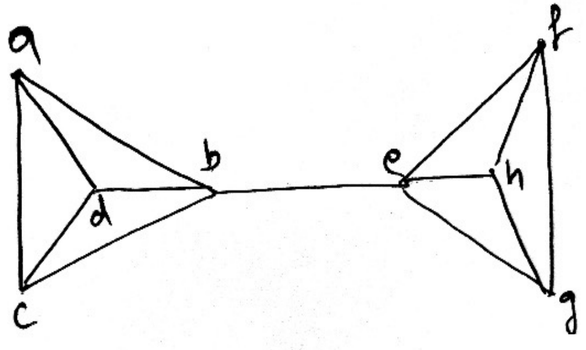
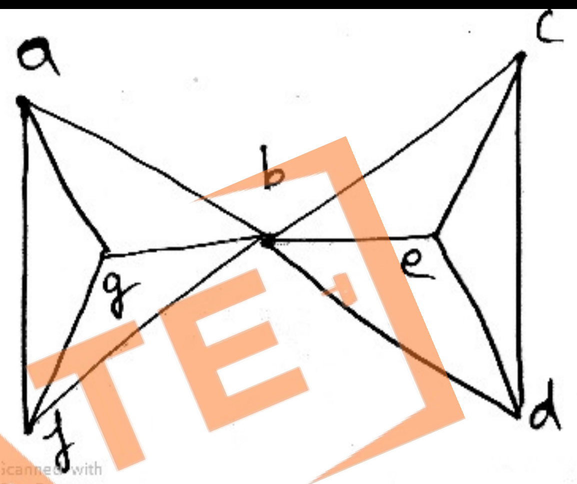
Connectivity: - each cut-set of a connected graph G consist of a certain number of edges. The number of edges in the smallest cut-set is defined as the edges connectivity of G. It is denoted by λ(G).

- if the edge connectivity from a graph is one, then that edge how's removal disconnect the graph is called a bridge.









k(G)				
λ(G)				
δ(G)				

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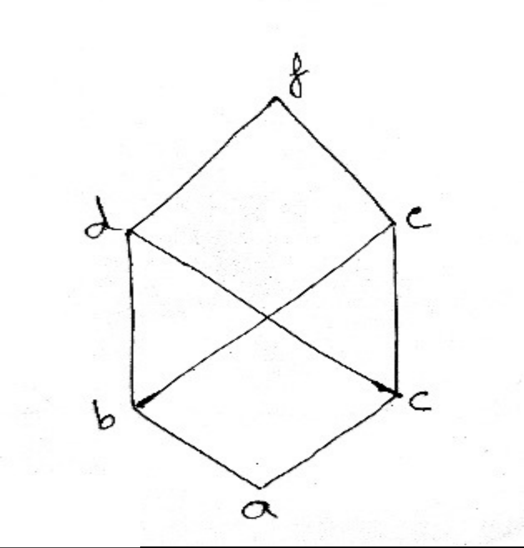
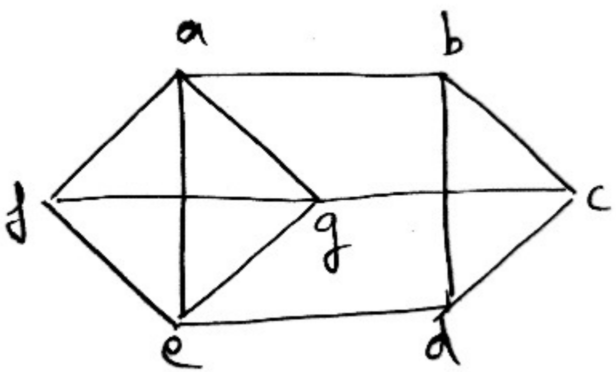
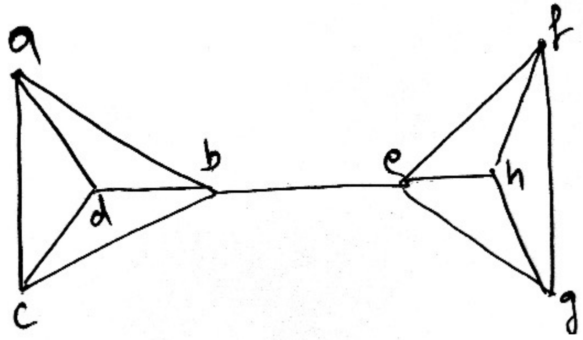
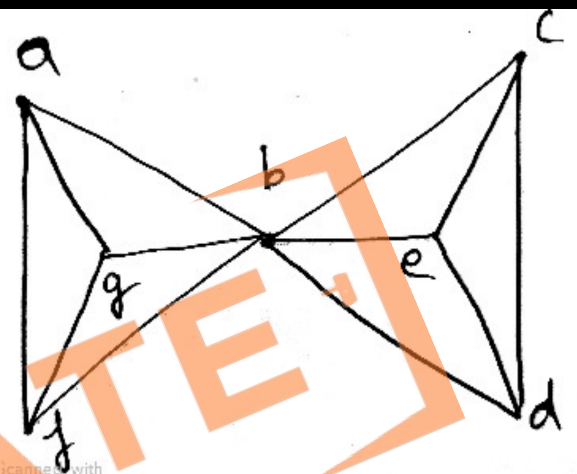
- Every Cut Set in a connected graph G must contain at least one branch of every spanning tree of G.
- Every circuit has an even number of edges in common with any Cut-Set.







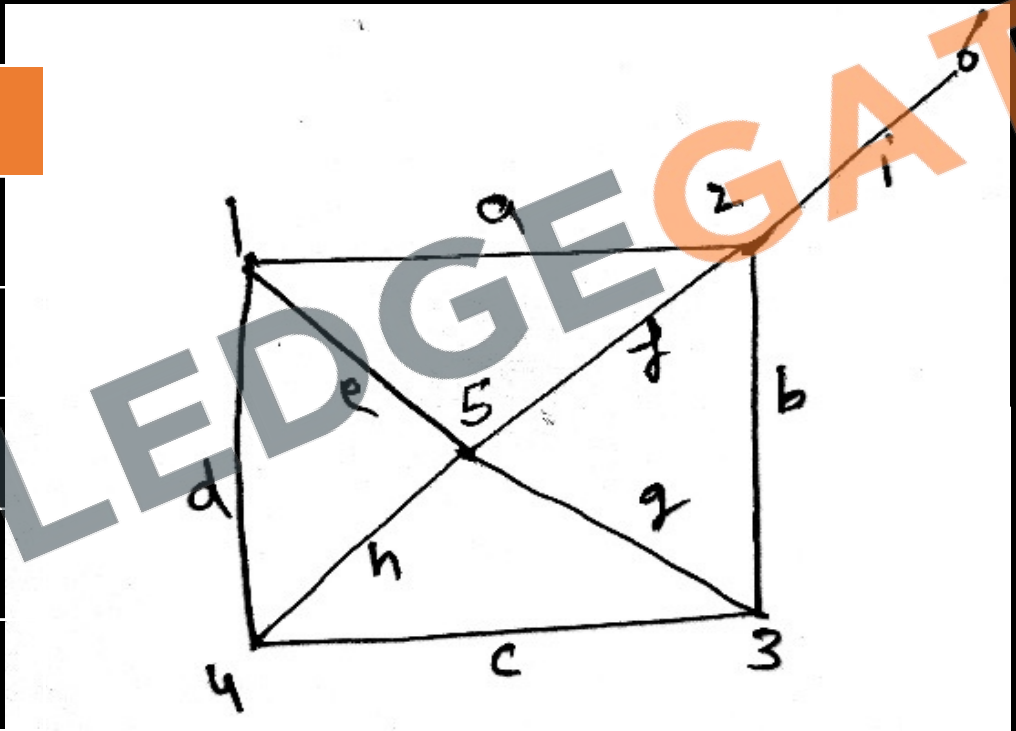


k(G)				
λ(G)				
δ(G)				

Cut-Set (Vertex)

Cut Set: - In a connected graph G, a cut set is a set of vertices whose removal from g leaves G disconnected, provided removal of no proper subset of these vertices disconnects G.

Cut Set	Validity
{5, 3}	
{6}	
{5, 2}	
{2}	
{1, 5, 3}	

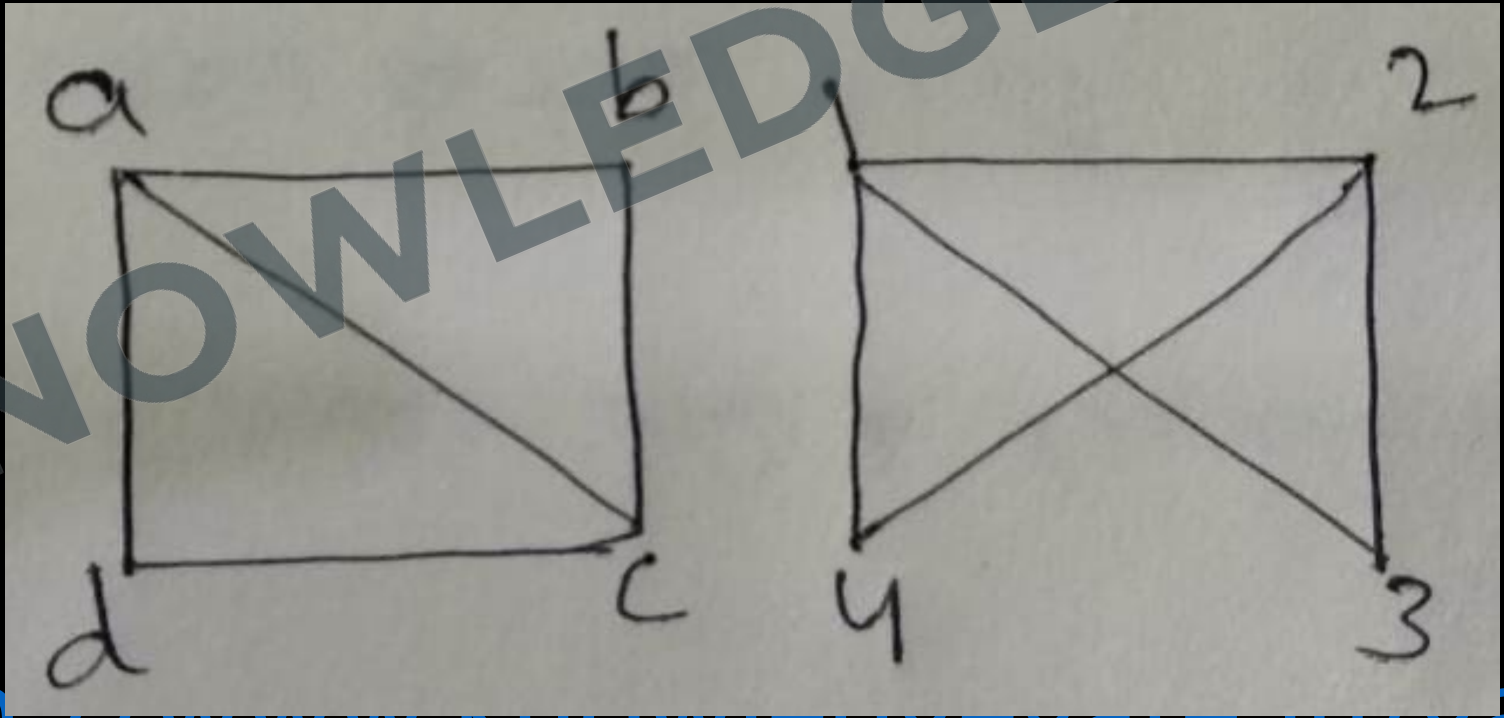


Vertex Connectivity: - Each cut-set of a connected graph G consist of a certain number of vertices. The number of vertices in the smallest cut-set is defined as the vertex connectivity of G. It is denoted by k(G).

- A connected graph is said to be separable of its vertex connectivity is one.
- If the vertex connectivity of a graph is one, then that vertex how removal disconnects a graph is called articulation point.

Isomorphism

1. In general, two graphs are said to be isomorphic if they are perhaps the same graphs, but just drawn differently with different names. i.e. two graphs are thought of as isomorphic if they have identical behavior in terms of graph-theoretic properties.
2. Formally speaking: - Two graphs G and G' are said to be isomorphic, if there is a one to one correspondence between their vertices and between their edges such that the incidence relationship is preserved.



1. Determining if two graphs are isomorphic is thought to be neither an NP-complete problem nor a P-problem, although this has not been proved.

2. In fact, there is a famous complexity class called graph isomorphism complete which is thought to be entirely disjoint from both NP-complete and from P.

Q How many simple non isomorphic graphs are possible with 3 vertices ?

Q How many simple non isomorphic graphs are possible with 4 vertices and 2 edges ?

Q How many simple non isomorphic graphs are possible with 4 vertices and 3 edges ?

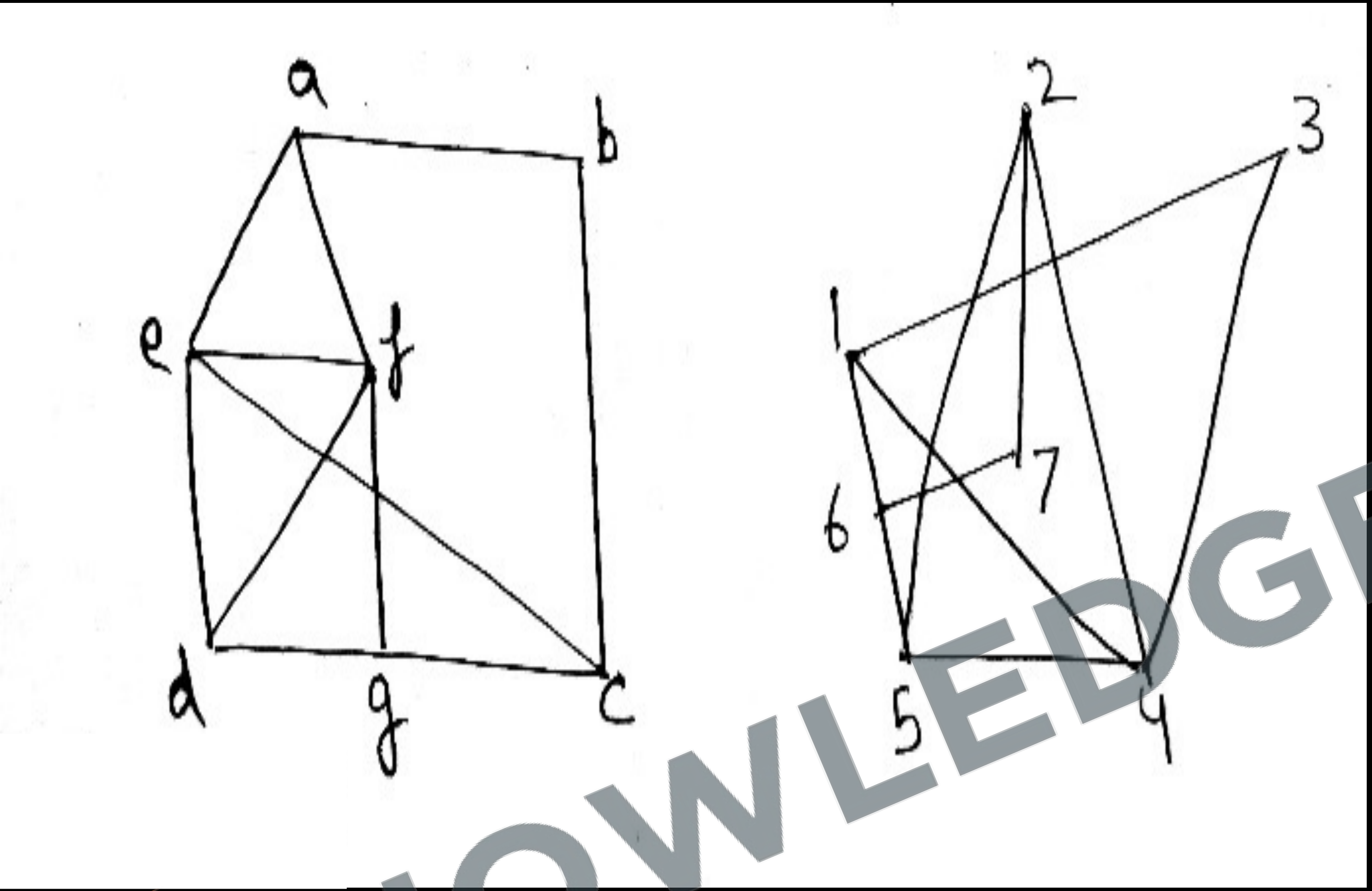
Q How many simple non isomorphic graphs are possible with 5 vertices and 3 edges ?

Q How many simple non isomorphic graphs are possible with 6 vertices and 6 edges, such that degree of every vertex must be same ?

Q How many simple non isomorphic graphs are possible with 8 vertices and 8 edges, such that degree of every vertex must be same ?

How to check weather two graphs are isomorphic or not

1. Number of vertices
2. Number of edges
3. Number of vertices with a given degree
4. Check degree property of vertices with their neighbor
5. Check minimum cycle length, maximum cycle length, or number of cycle with a specific length
6. Can check isomorphism for complement of the graph
7. Planer, non-planer
8. Connected disconnected
9. Chromatic number
10. Matching number, covering number
11. Edge connectivity, vertex connectivity
12. If it seems that graphs are isomorphic to each other then identify the similar vertex and delete both, and keep repeating the process until we are sure..



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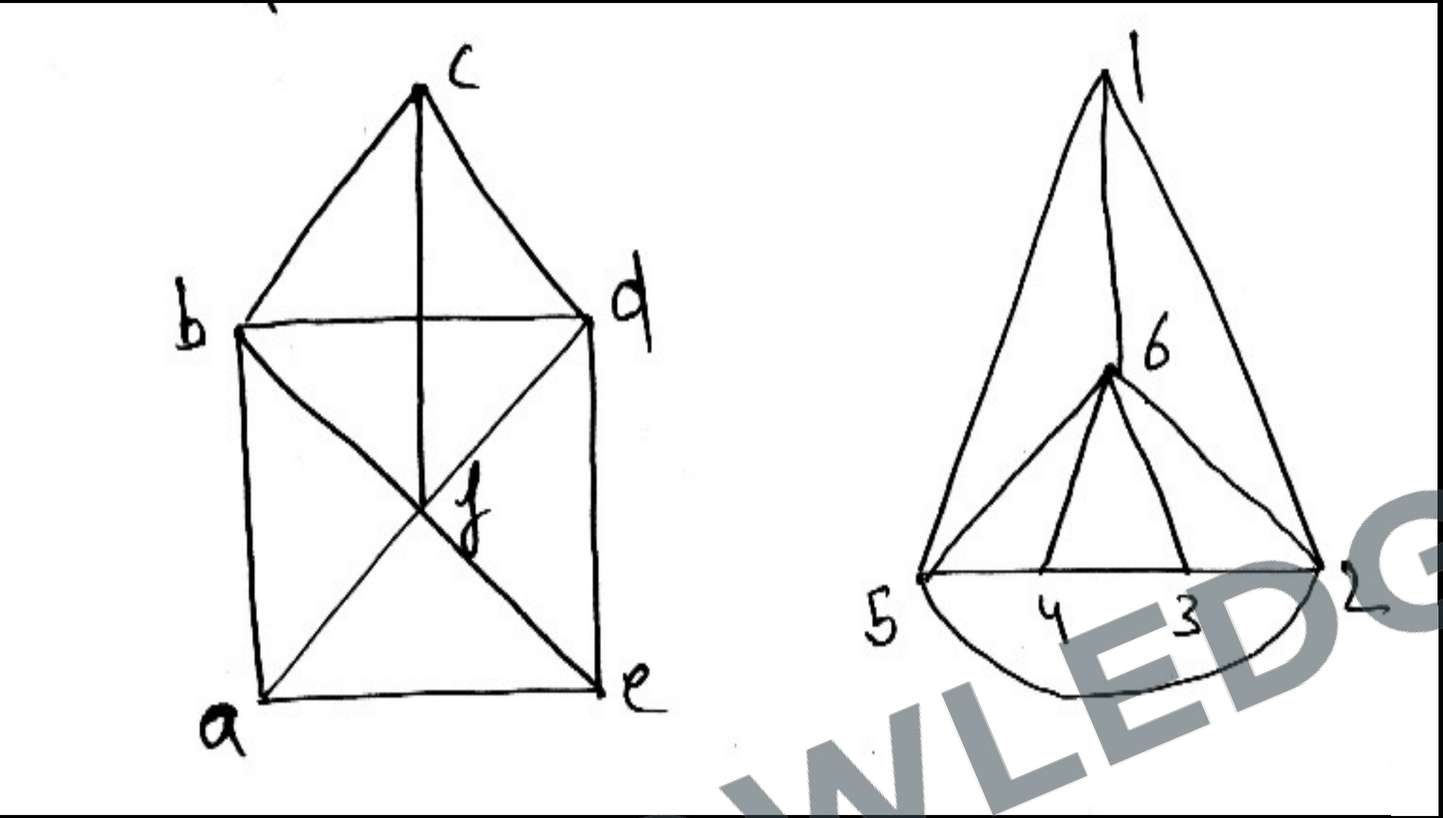
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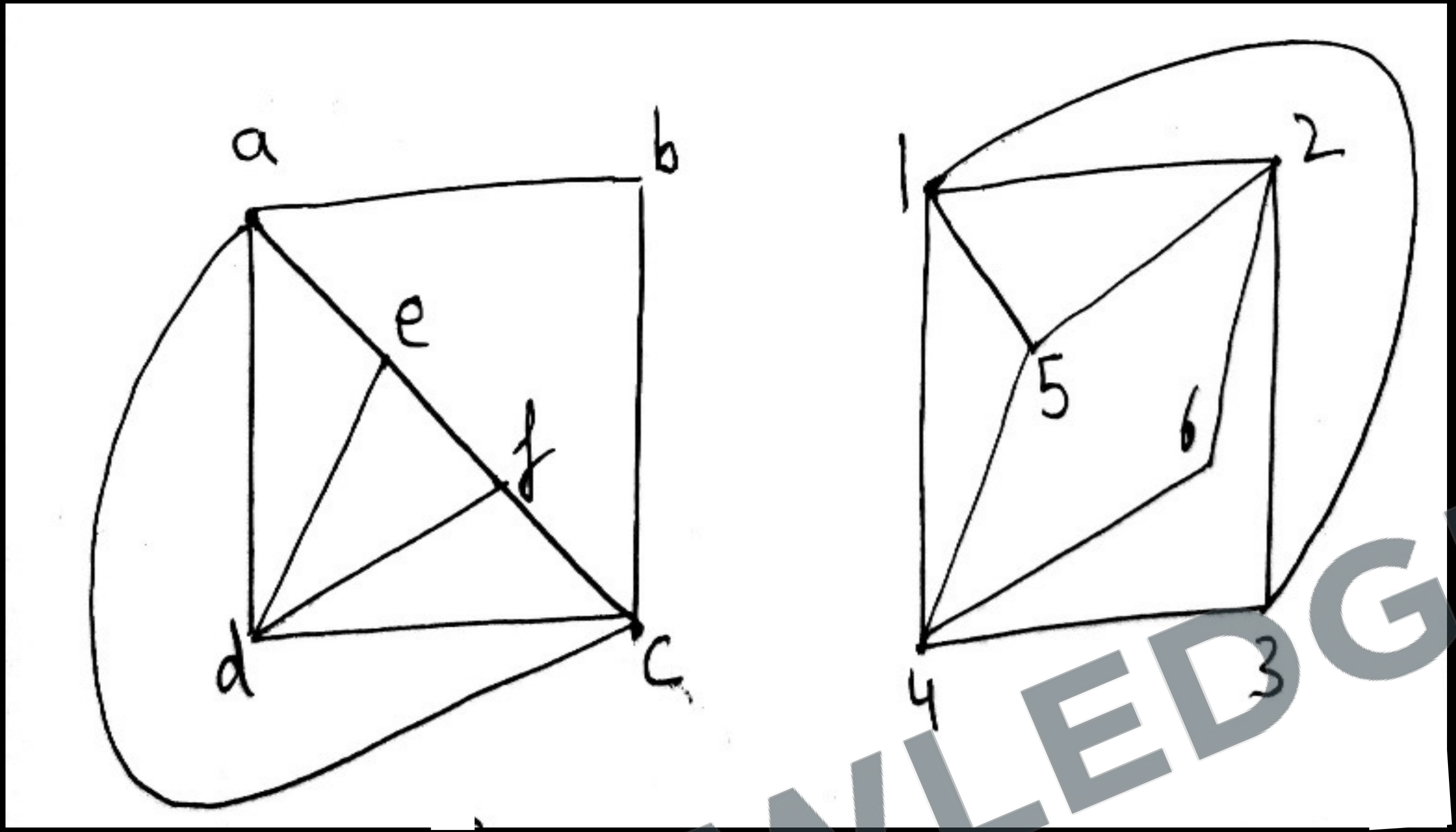
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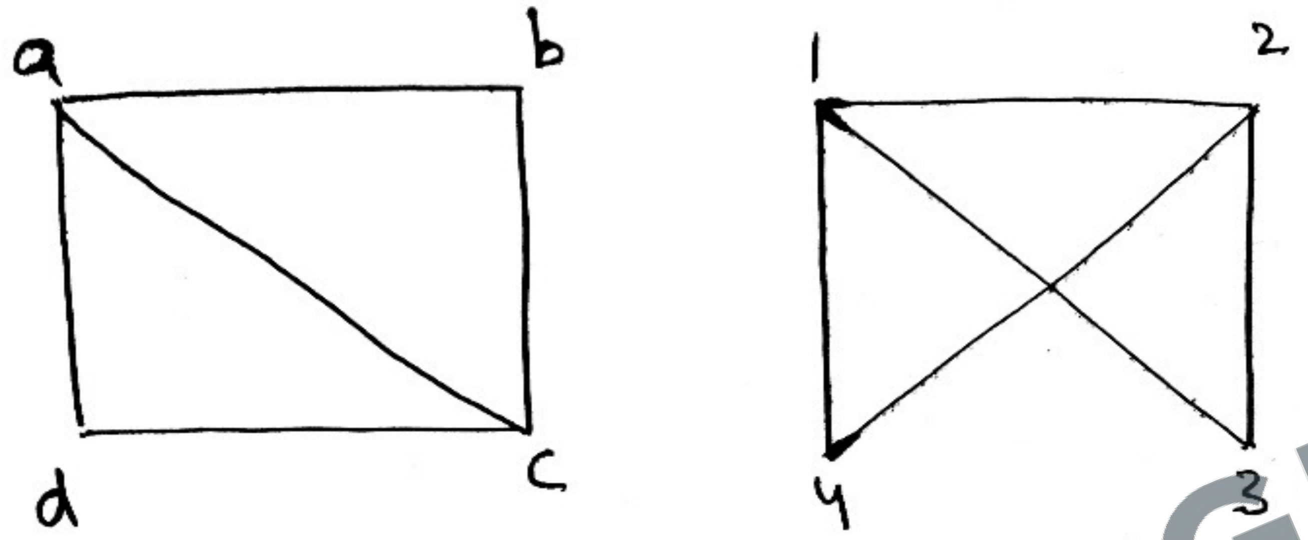
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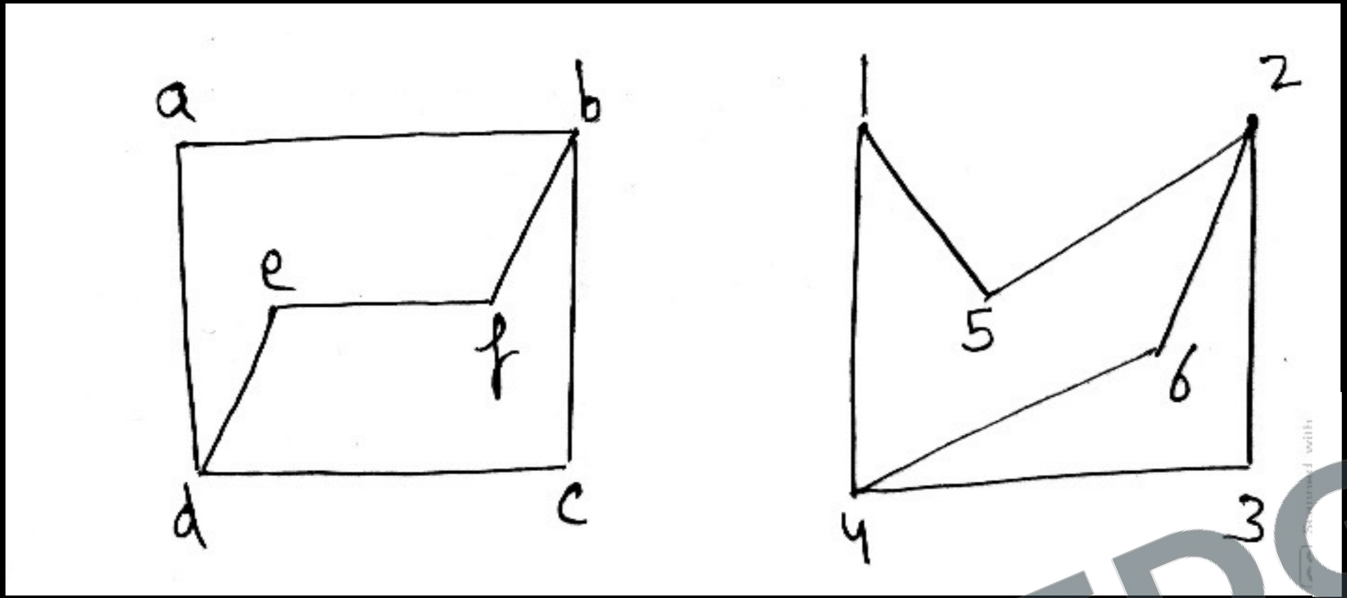

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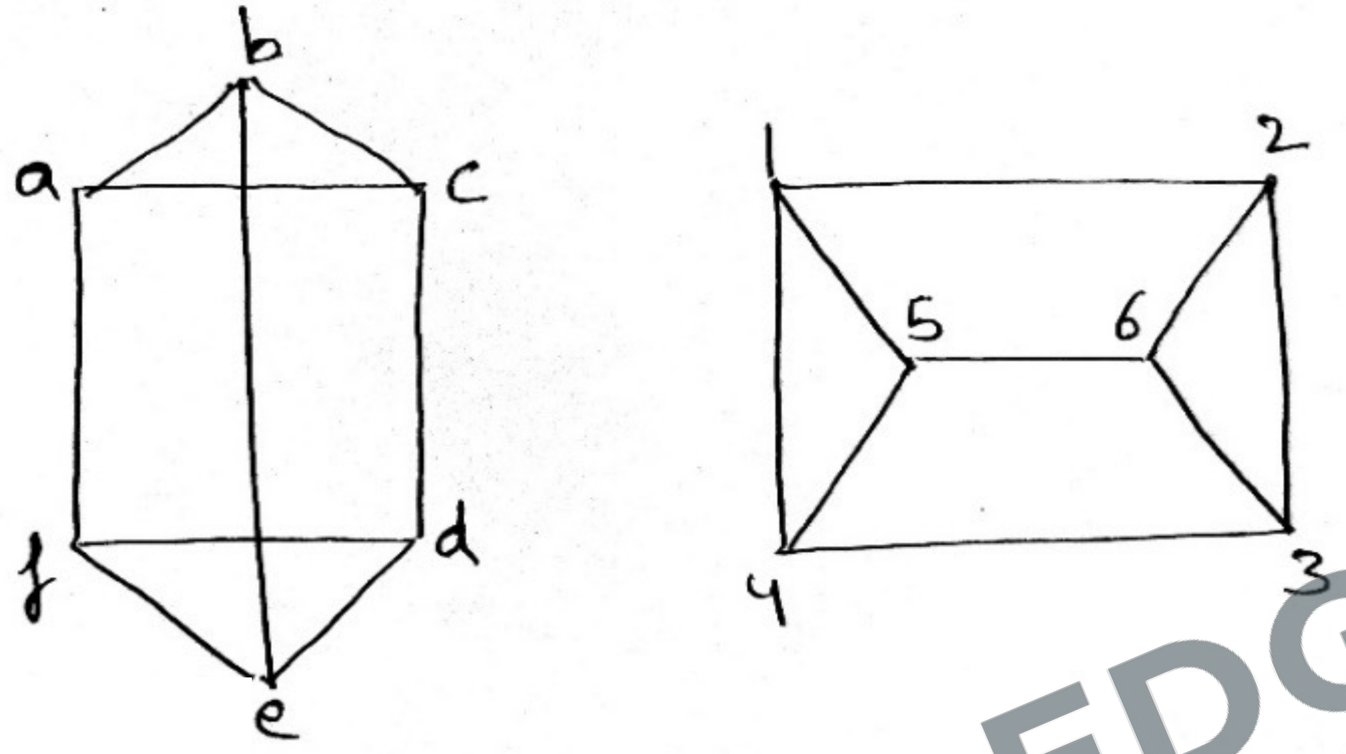
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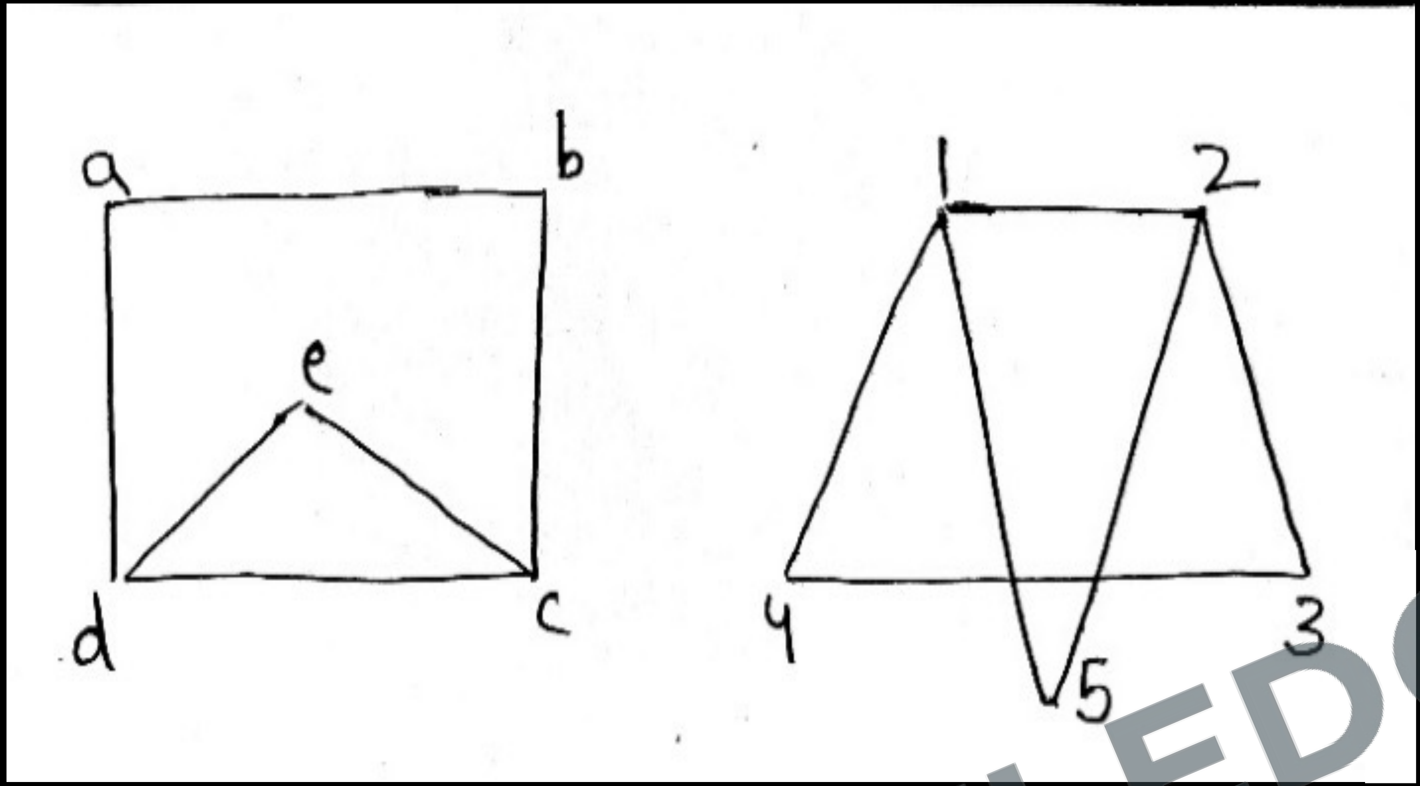

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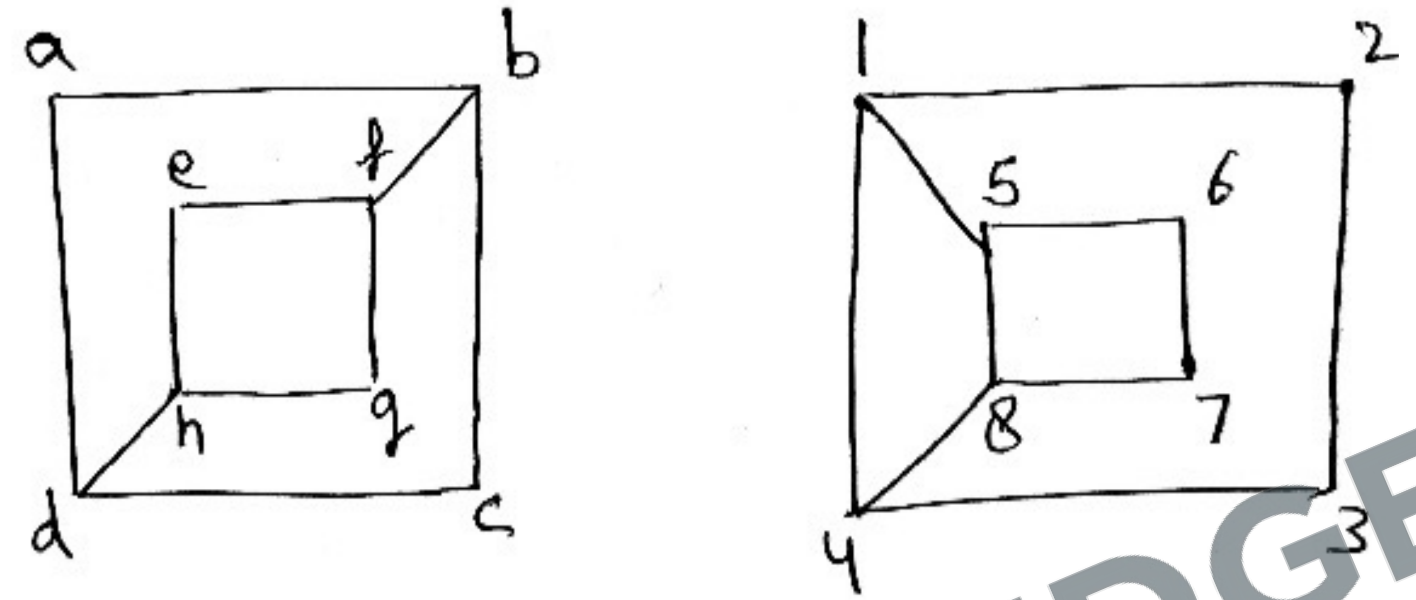
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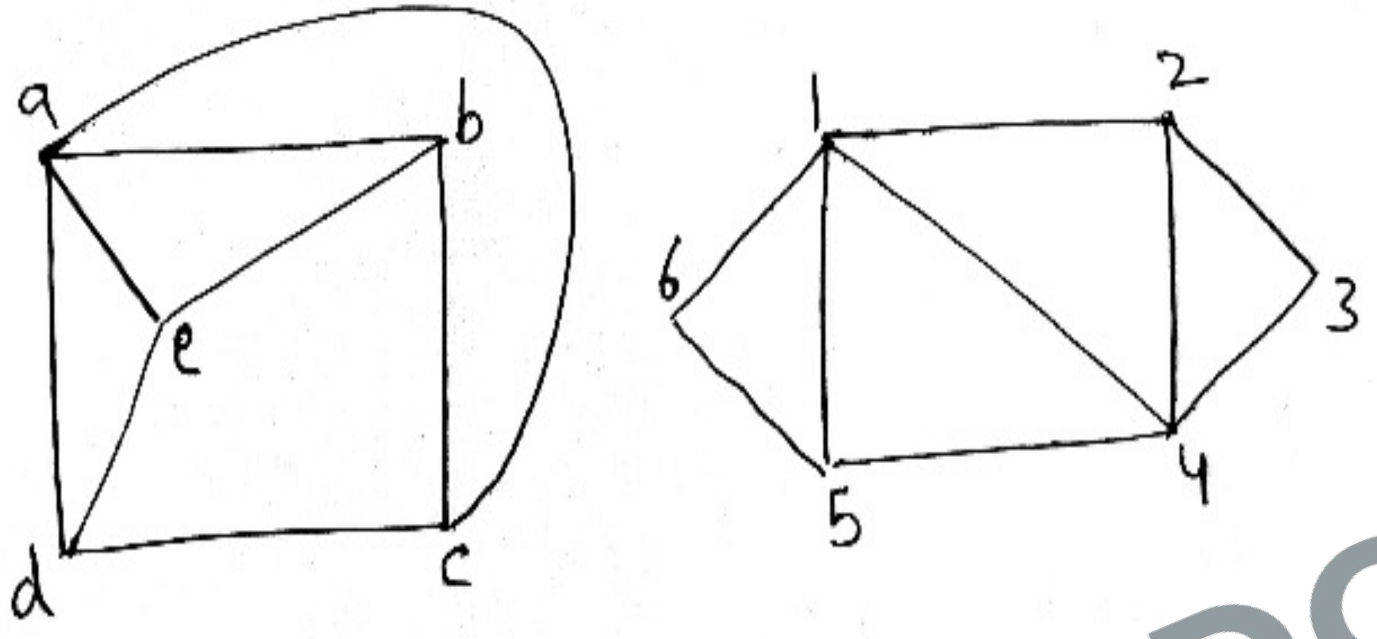
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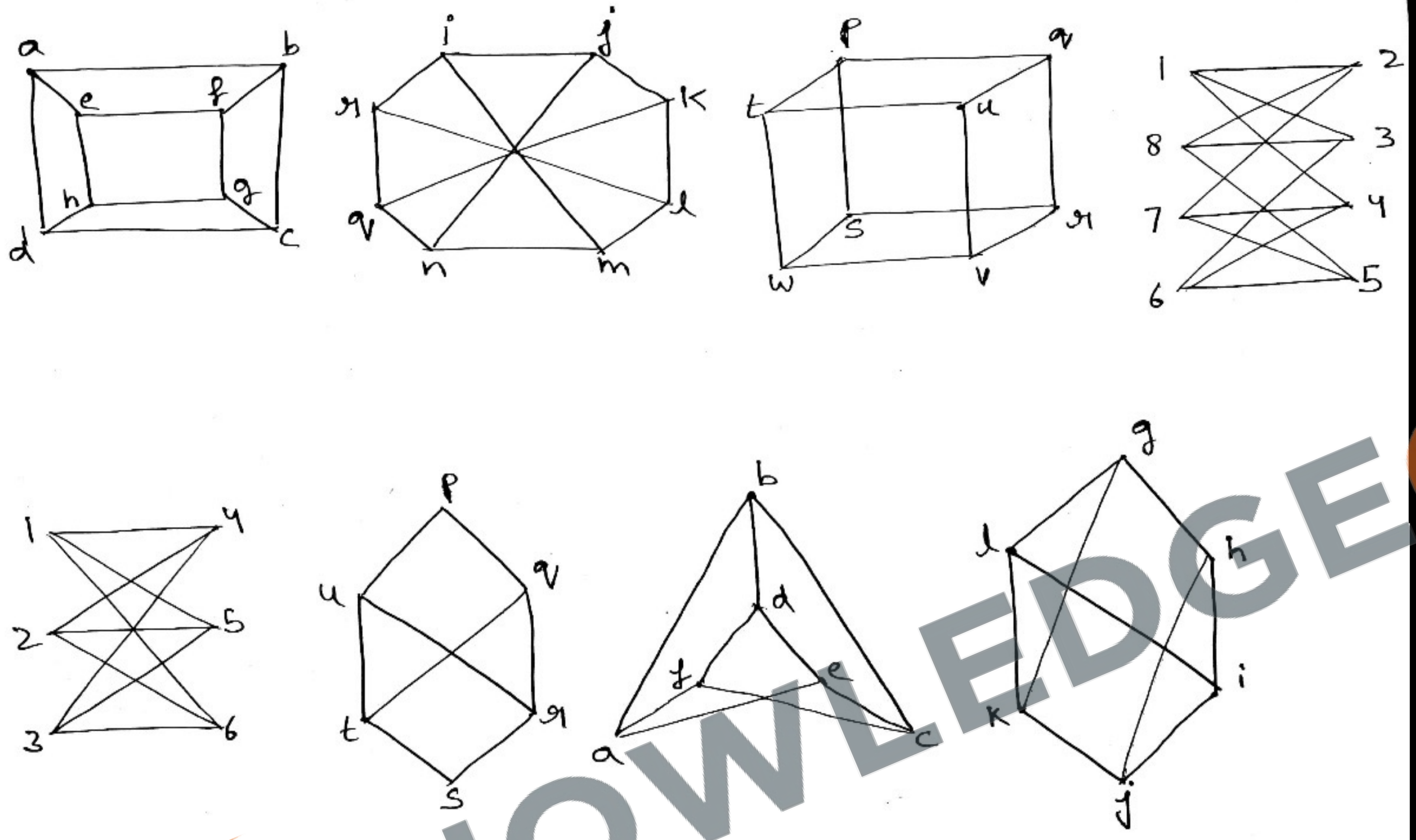

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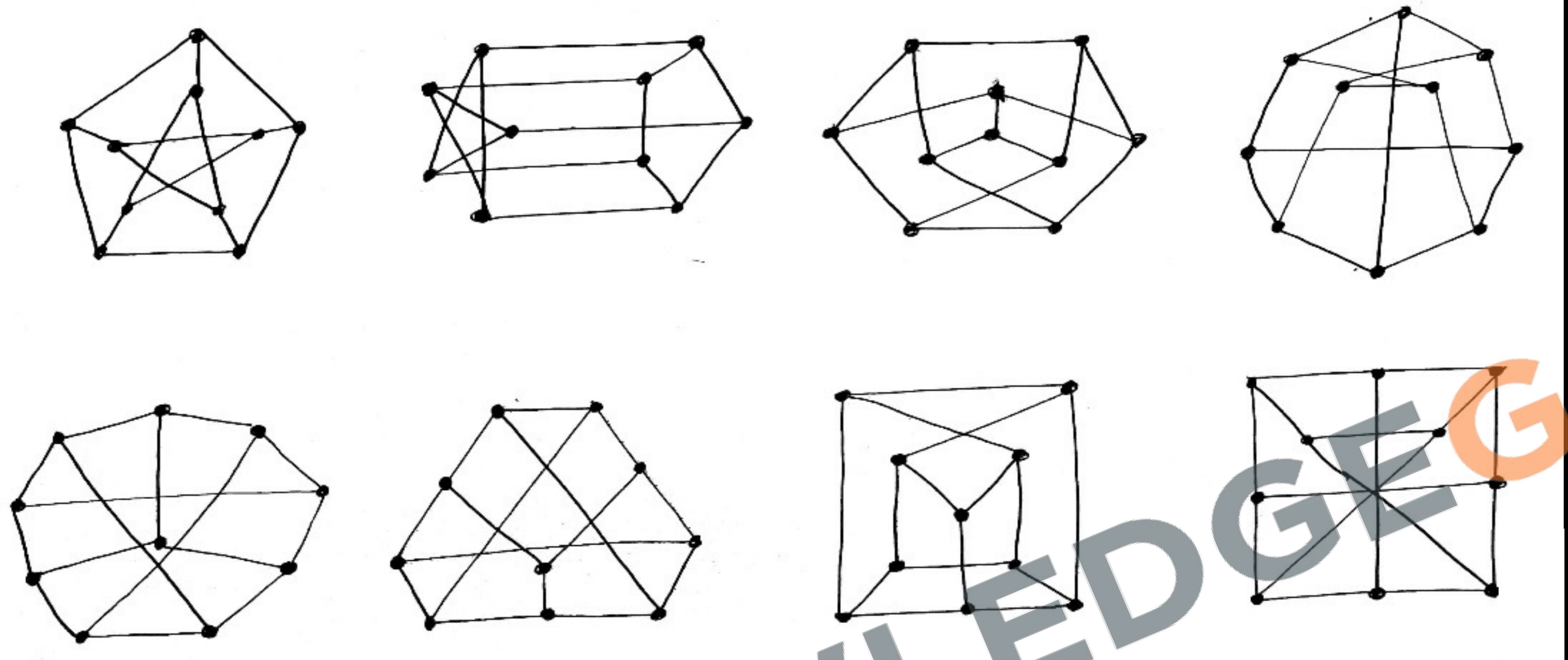
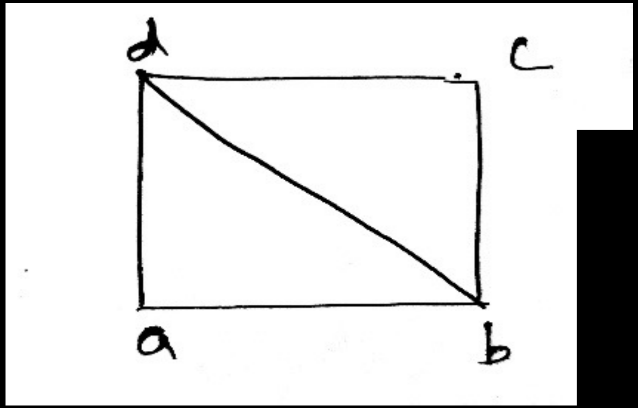
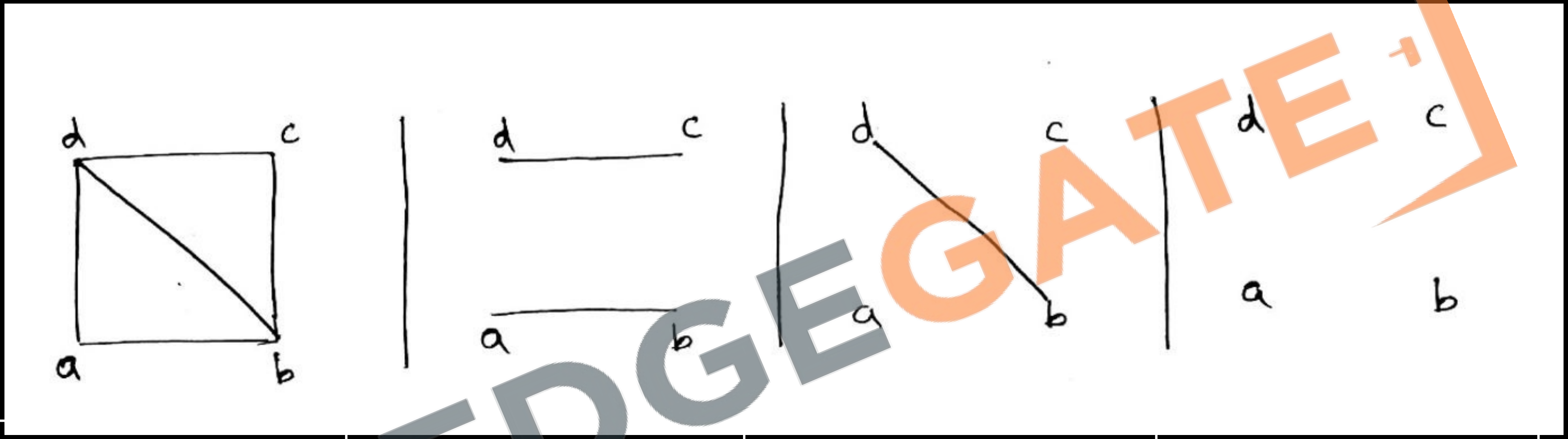


Matching: - Let G be a graph, a subgraph M of G is called a matching of G, if every vertex of G is incident with at most one edge in M.

$$deg(v) <= 1, \forall_v V \in V(G)$$





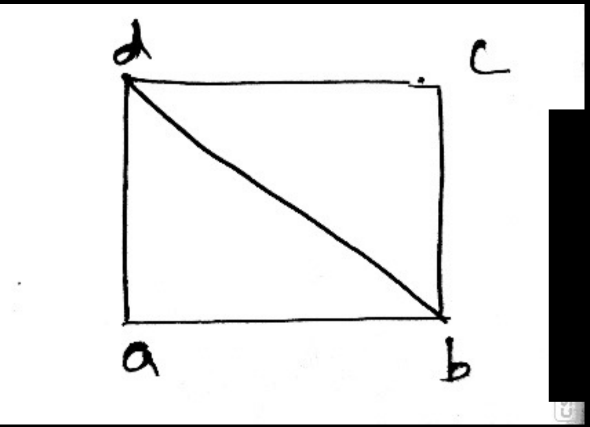
M_1	M_2	M_3	M_4

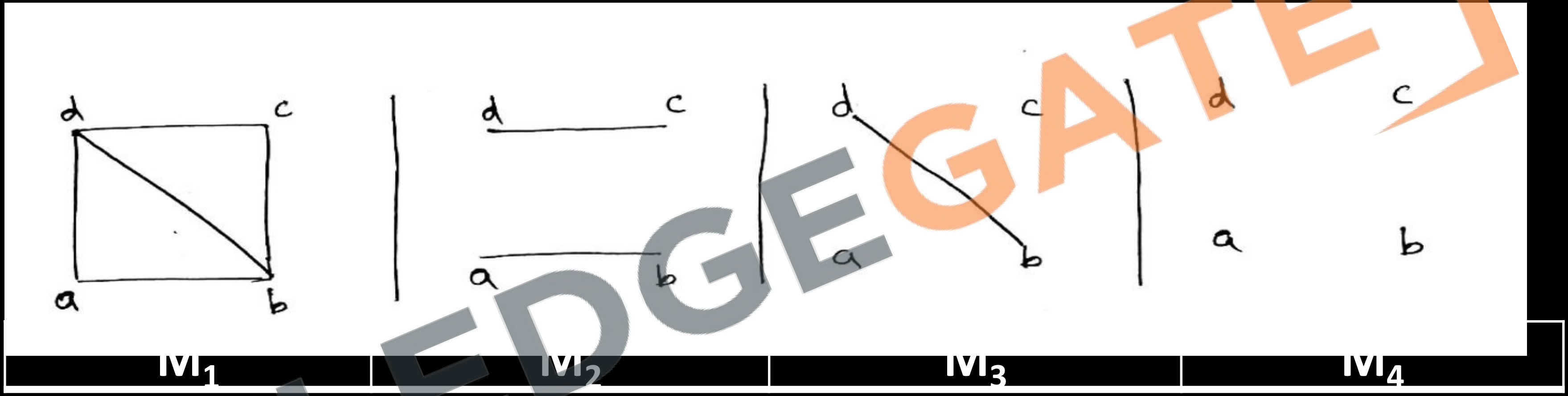
- In matching no two edges are adjacent

Maximal Matching: - A matching M of a graph G is said to be maximal, if no other edges of G can be added to M, without violating the deg condition.

Maximum Matching: - A matching of a graph with maximum no of edges is called a maximum matching of G.

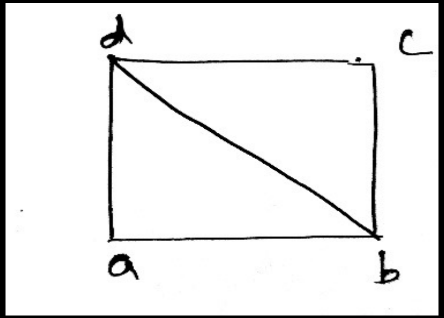
- Number of edges in a maximum matching of G is called matching number.

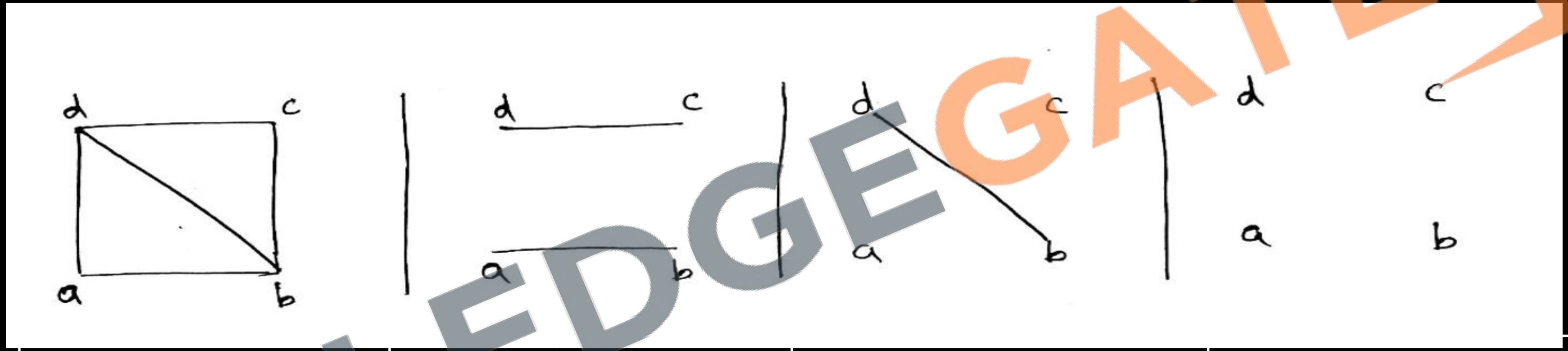




Perfect Matching: - A matching of a graph in which every vertex is matched is called perfect matching.

1. If a graph G has a perfect match then no of vertices in G is even.
2. If no of vertexes is even, it is not necessary to have a perfect match.
3. No of perfect matchings are there in a complete graph K_n is $[(2n)!]/n!2^n$



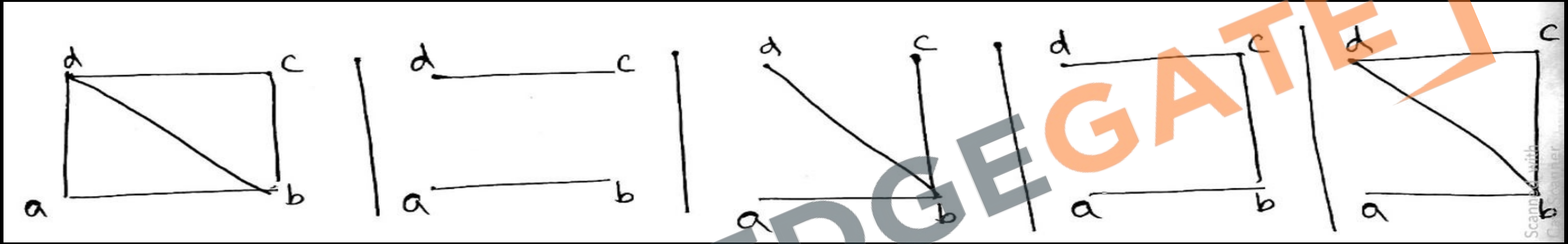


M_1	M_2	M_3	M_4

Covering

Line Covering: - Let G (V, E) be a graph, a subset C of E is called a line covering of G, if every vertex of G is incident with at least one edge in C. (deg at least one)

deg(v) >= 1



C_1	C_2	C_3	C_4

- Line covering of a graph G does not exist if G has an isolated vertex

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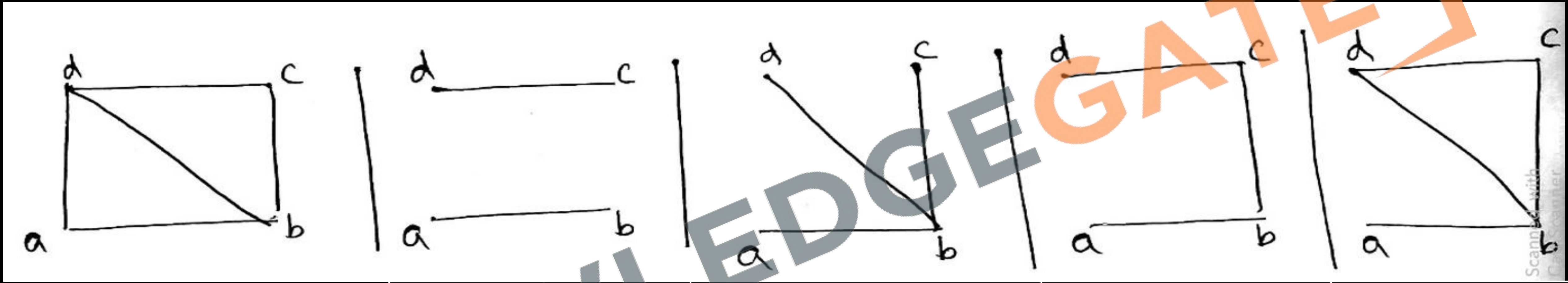
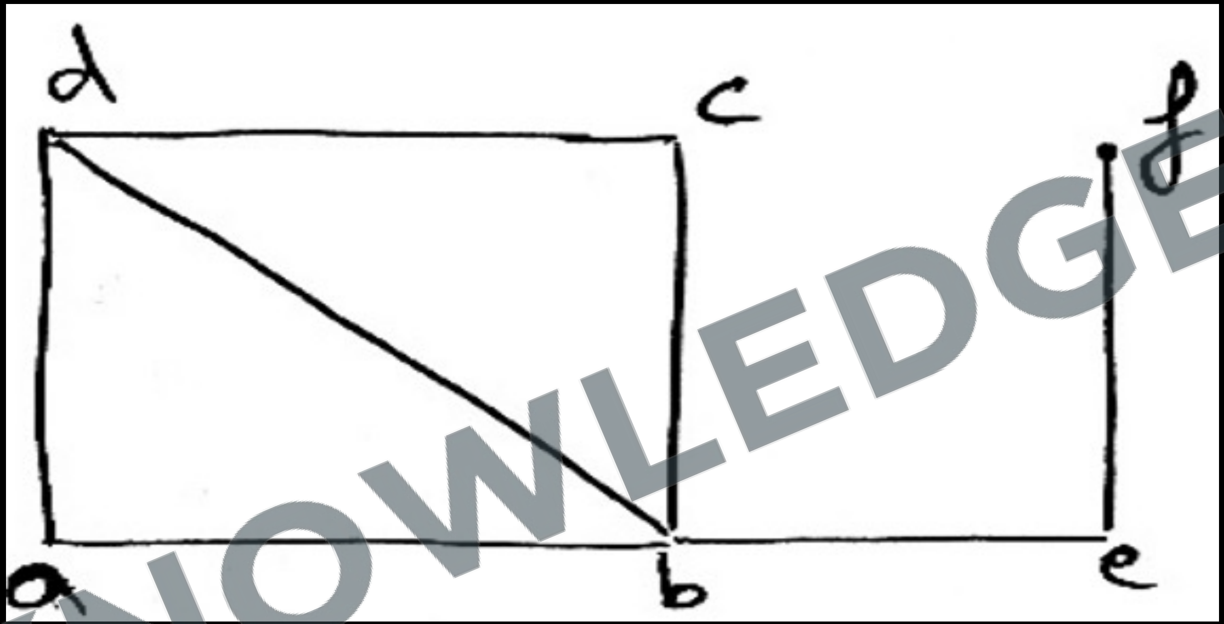
- **Minimal Line covering:** - A line covering is said to be minimal if no edge can be deleted from the line covering, without destroying its ability to cover the graph.
- **Minimum line covering:** - A line covering with minimum no of edges is called a minimum line covering.



C_1	C_2	C_3	C_4

- No of edges in minimum line covering is called **line covering number** of a graph G, denoted by α_1
- line covering of a graph with n vertices contain at least upper bound(n/2) edges.
- no minimal line covering can contain a cycle.



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Independent Line set: - Let G (V, E) be a graph, a subset L of E is called independent line set of G, if no two edges are adjacent.

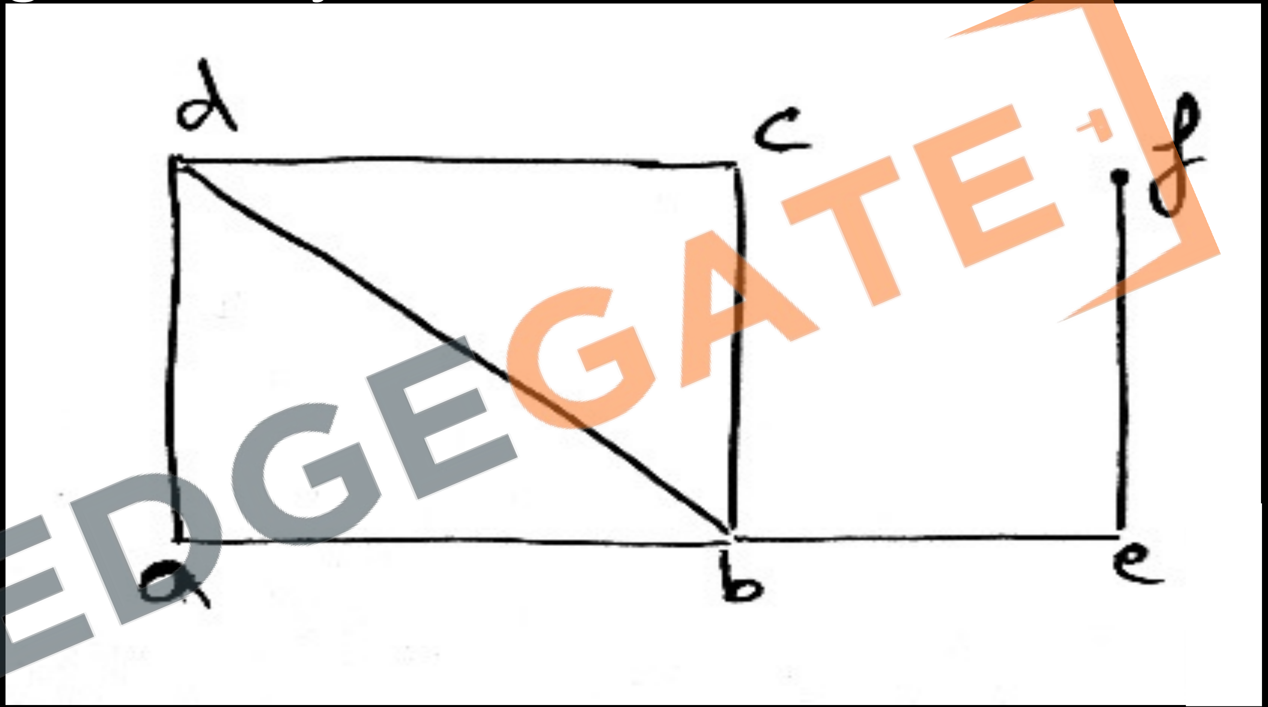
$L_1 = \{(b, d)\}$

$L_2 = \{(b, d), (e, f)\}$

$L_3 = \{(a, d), (b, c), (e, f)\}$

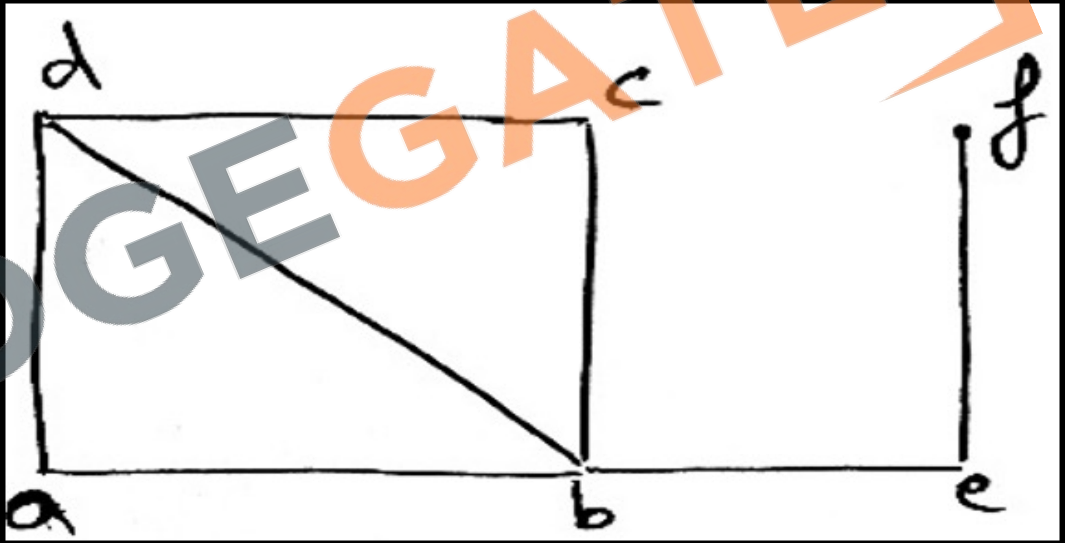
$L_4 = \{(a, b), (e, f)\}$

$L_5 = \{(a, b), (d, c), (e, f)\}$



- **<u>Maximal independent Line set:</u>** - An independent line set L of a graph G is said to be maximal if no other edges of G can be added to L.
- **<u>Maximum independent line set:</u>** - An independent line set L of a graph G, with maximum no of edges is called maximum independent line set.
- No of edges in maximum independent line set is called **Line independent number** of G denoted by β_1.
- line independent no = matching no of G

$$\alpha_1 + \beta_1 = |v|$$



$L_1 = \{(b, d)\}$

$L_2 = \{(b, d), (e, f)\}$

$L_3 = \{(a, d), (b, c), (e, f)\}$

$L_4 = \{(a, b), (e, f)\}$

$L_5 = \{(a, b), (d, c), (e, f)\}$

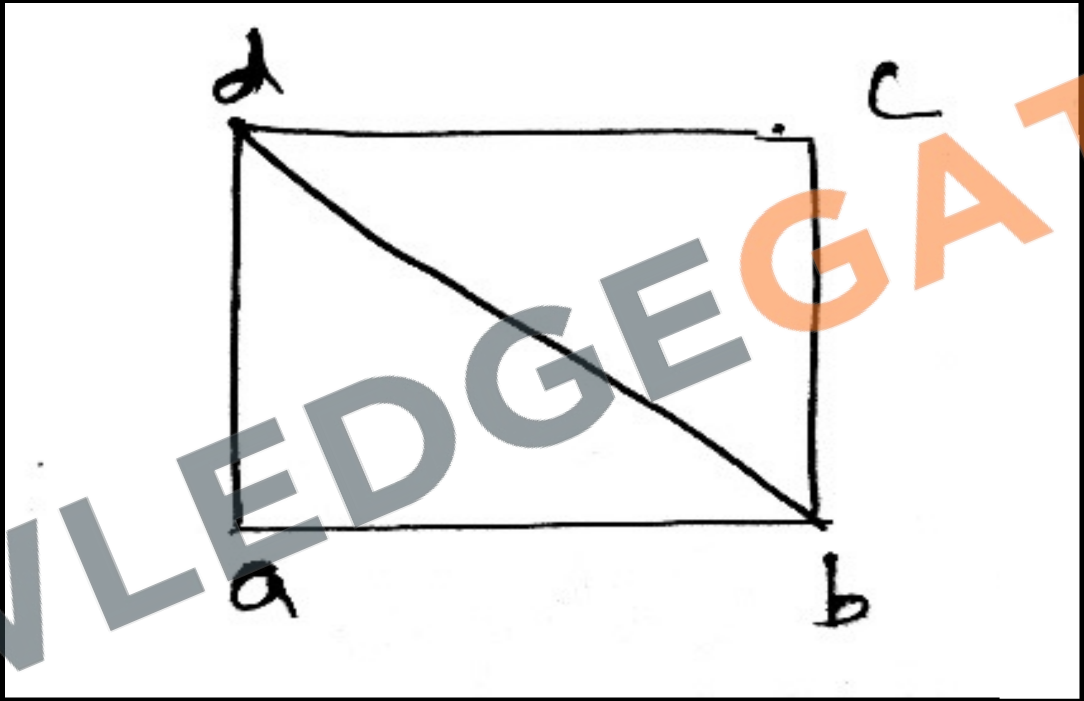
Vertex Covering: - Let G (V, E) be graph, a subset K of V is called a vertex coving of G. if every edge of G is incident with a vertex in K.

$K_1 = \{b, d\}$

$K_2 = \{a, b, c\}$

$K_3 = \{b, c, d\}$

$K_4 = \{a, b, c, d\}$



Minimal vertex cover: - Vertex covering K of a graph G is said to be minimal if no vertex can be deleted from K, without violating the condition.

Minimum vertex covering: - A vertex covering of a graph G with minimum number of vertices is called as minimum vertex covering.

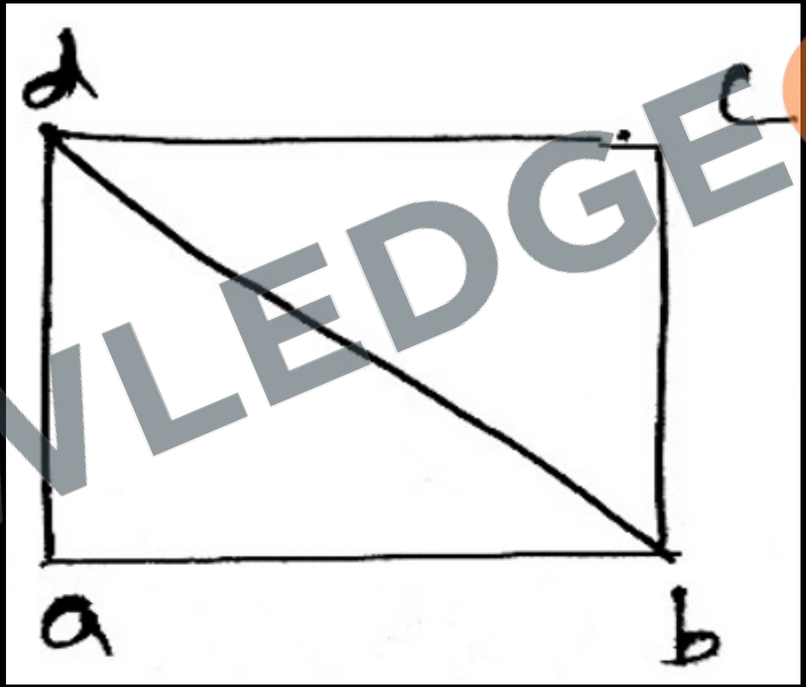
- No of vertices in a minimum vertex covering is called **vertex Covering no** of graph G denoted by α_2

$K_1 = \{b, d\}$

$K_2 = \{a, b, c\}$

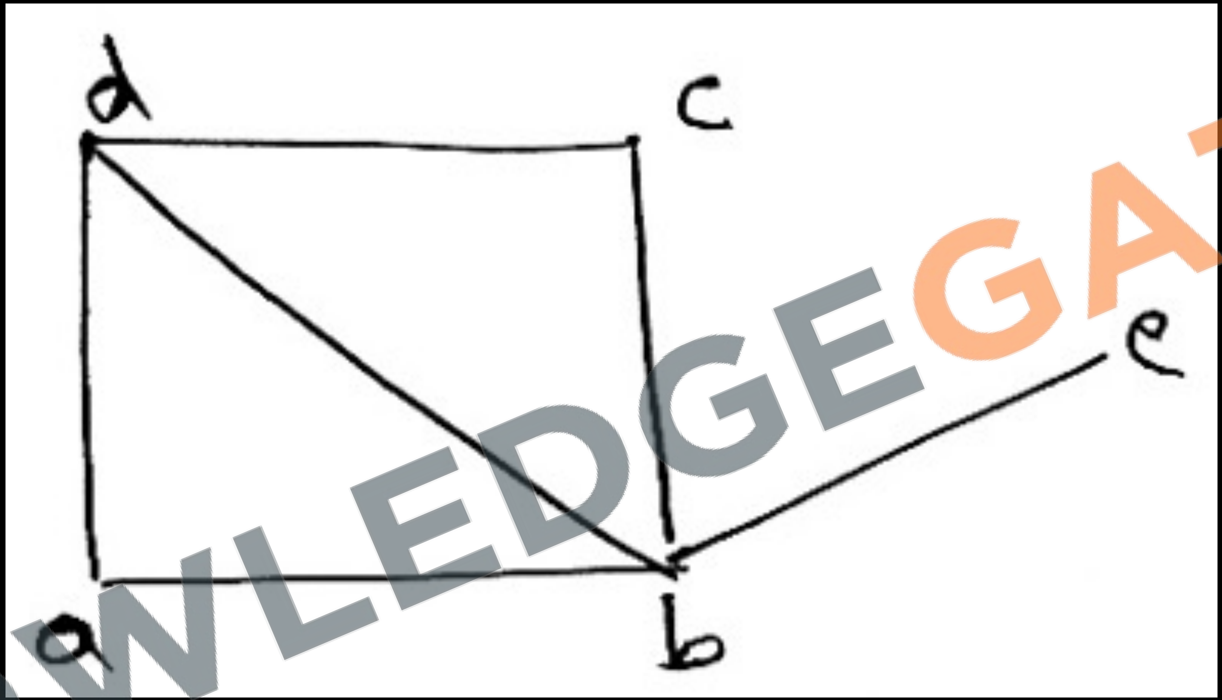
$K_3 = \{b, c, d\}$

$K_4 = \{a, b, c, d\}$



Independent vertex set: - let G (V, E) be a graph, a subset S of V is called an independent vertex set if no two vertices in S are adjacent.

$S_1 = \{b\}$
$S_2 = \{d, e\}$
$S_3 = \{a, c\}$
$S_4 = \{a, b, c\}$
$S_5 = \{a, c, e\}$



Maximum independent Vertex Set: - An independent vertex set is said to be maximal, if no other vertex of G can be added to the set.

Maximum independent vertex set: - An independent vertex set of graph G with maximum no of vertices is called maximum independent vertex set.

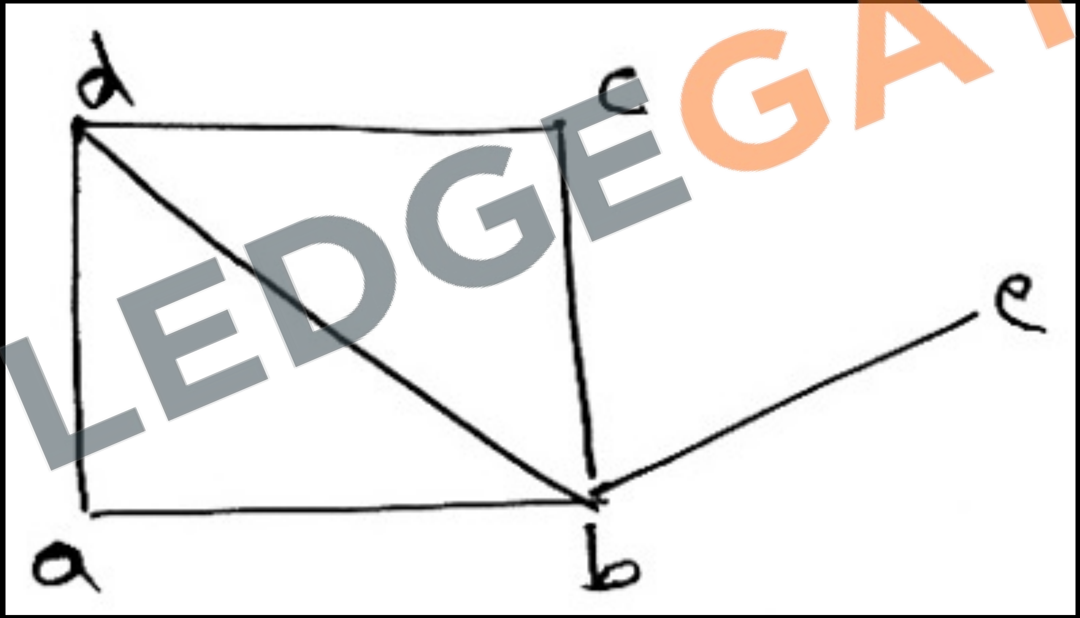
The number of vertices in maximum independent vertex set is called as **vertex independent number** of G donated by β_2

$\alpha_2 + \beta_2 = |V|$

$S_1 = \{b\}$

$S_2 = \{d, e\}$

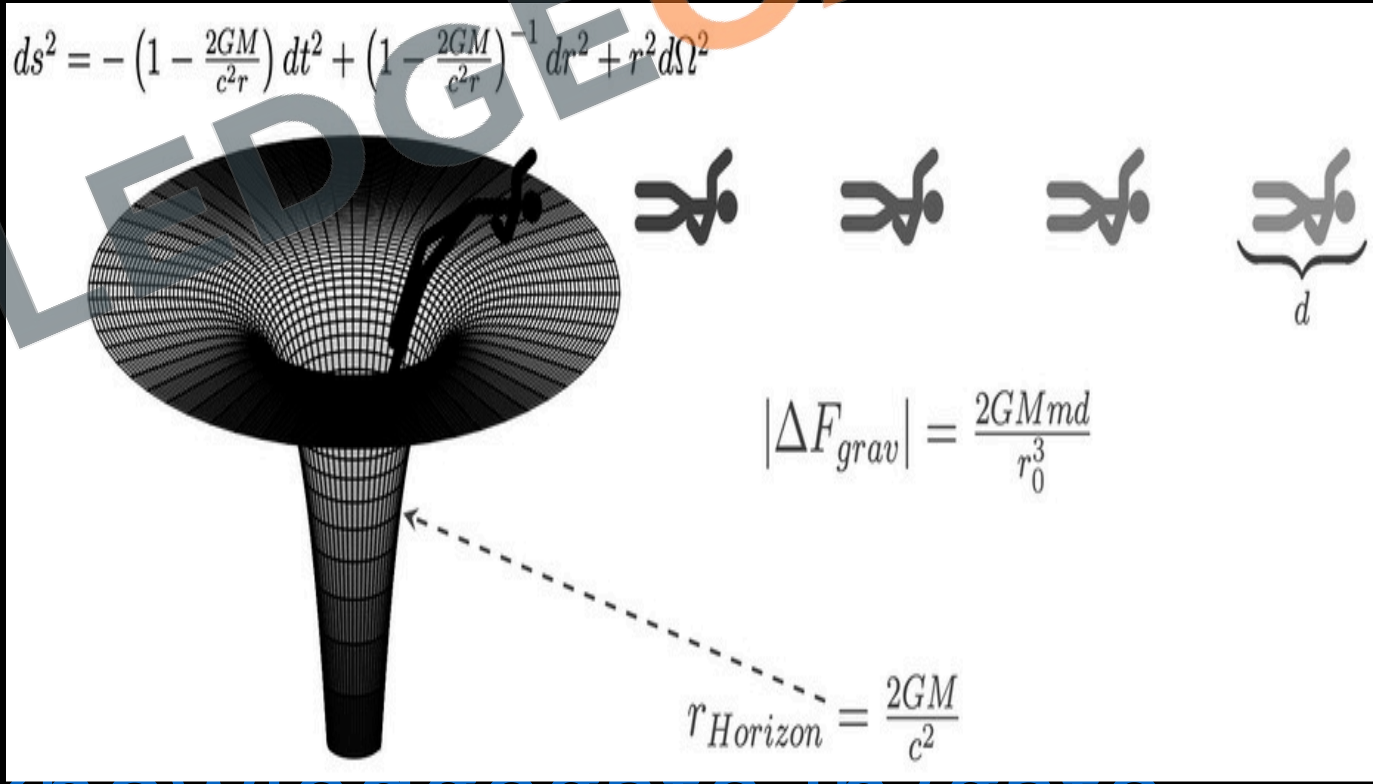
$S_3 = \{a, c, e\}$



•Chapter-6 (Group Theory):

Group Theory

- Group theory is very important mathematical tool which is used in a number of areas in research and application. Using group theory, we can estimate the strength of a set with respect to an operator. This idea will further help us in research field to identify the correct mathematical system to work in a particular research area. E.g. can we use natural numbers in complex problem area like soft computing or studying black holes.



1. **Closure property: -** Consider a non-empty set A and a binary operation * on A. A is said to be closed with respect to *, if ∀ a, b ∈ A, then a*b ∈ A.

2. **Algebraic Structure: -** A non-empty set A is said to be an algebraic structure with respect to a binary operation *, if A satisfy closure property with respect to *.

		Algebraic Structure
1	(N, +)	
2	(N, -)	
3	(N, /)	
4	(N, x)	
5	(Z, +)	
6	(Z, -)	
7	(Z, /)	
8	(Z, x)	
9	(R, +)	
10	(R, -)	
11	(R, /)	
12	(R, x)	
13	(M, +)	
14	(M, x)	
15	(E, +)	
16	(E, x)	
17	(O, +)	
18	(O, x)	
19	(R-0, x)	
20	(R-0, /)	
21	(Non-Singular Matrix, x)	

1. **Associative property: -** Consider a non-empty set A and a binary operation * on A. A is said to be associative with respect to *, if ∀ a, b, c ∈ A, then (a*b) *c = a*(b*c)

2. **Semi-Group: -** A non-empty set A is said to be a Semi-group with respect to a binary operation *, if A satisfy closure, Associative property with respect to *.

		Algebraic Structure	Semi Group
1	(N, +)	Y	
2	(N, -)	N	
3	(N, /)	N	
4	(N, x)	Y	
5	(Z, +)	Y	
6	(Z, -)	Y	
7	(Z, /)	N	
8	(Z, x)	Y	
9	(R, +)	Y	
10	(R, -)	Y	
11	(R, /)	N	
12	(R, x)	Y	
13	(M, +)	Y	
14	(M, x)	Y	
15	(E, +)	Y	
16	(E, x)	Y	
17	(O, +)	N	
18	(O, x)	Y	
19	(R-0, x)	Y	
20	(R-0, /)	Y	
21	(Non-Singular Matrix, x)	Y	

1. **Identity property: -** Consider a non-empty set A and a binary operation * on A. A is said to satisfy identity property with respect to *, if ∀ a ∈ A, there must be unique e ∈ A, such that
   a*e = e*a = a

2. There is exactly one Identity element in the set and will be same for all element in the set.

3. **Monoid: -** A non-empty set A is said to be a Monoid with respect to a binary operation *, if A satisfy closure, Associative, identity property with respect to *.

		Algebraic Structure	Semi Group	Monoid
1	(N, +)	Y	Y	
2	(N, -)	N	N	
3	(N, /)	N	N	
4	(N, x)	Y	Y	
5	(Z, +)	Y	Y	
6	(Z, -)	Y	N	
7	(Z, /)	N	N	
8	(Z, x)	Y	Y	
9	(R, +)	Y	Y	
10	(R, -)	Y	N	
11	(R, /)	N	N	
12	(R, x)	Y	Y	
13	(M, +)	Y	Y	
14	(M, x)	Y	Y	
15	(E, +)	Y	Y	
16	(E, x)	Y	Y	
17	(O, +)	N	N	
18	(O, x)	Y	Y	
19	(R-0, x)	Y	Y	
20	(R-0, /)	Y	N	
21	(Non-Singular Matrix, x)	Y	Y	

1. **Inverse property: -** Consider a non-empty set A and a binary operation * on A. A is said to satisfy inverse property with respect to *, if ∀ a ∈ A, there must be unique element a^{-1} ∈ A, such that
   $a * a^{-1} = a^{-1} * a = e$

2. Every element has a exactly one unique inverse which is also present in the same set.

3. If a is the inverse of b, then b will be invers one a.

4. No two elements can have the same inverse

5. Identity element is its own inverse.

6. **Group: -** A non-empty set A is said to be a group with respect to a binary operation *, if A satisfy closure, Associative, identity, inverse property with respect to *.

		AS	Semi Group	Monoid	Group
1	(N, +)	Y	Y	N	
2	(N, -)	N	N	N	
3	(N, /)	N	N	N	
4	(N, x)	Y	Y	Y	
5	(Z, +)	Y	Y	Y	
6	(Z, -)	Y	N	N	
7	(Z, /)	N	N	N	
8	(Z, x)	Y	Y	Y	
9	(R, +)	Y	Y	Y	
10	(R, -)	Y	N	N	
11	(R, /)	N	N	N	
12	(R, x)	Y	Y	Y	
13	(M, +)	Y	Y	Y	
14	(M, x)	Y	Y	Y	
15	(E, +)	Y	Y	Y	
16	(E, x)	Y	Y	N	
17	(O, +)	N	N	N	
18	(O, x)	Y	Y	Y	
19	(R-0, x)	Y	Y	Y	
20	(R-0, /)	Y	N	N	
21	(Non-Singular Matrix, x)	Y	Y	Y	

1. If the total number of elements in a group is even then there exists at least one element in the group who is the inverse of itself.

2. Some time it is also possible that every element is inverse of itself in a group.

3. In a group $(a * b)^{-1} = b^{-1} * a^{-1}$ for $\forall$ a, b $\in$ A

4. Cancelation law holds good

    1. $a * b = a * c \rightarrow b = c$

    2. $a * c = b * c \rightarrow a = b$

1. **Commutative property: -** Consider a non-empty set A and a binary operation * on A. A is said to satisfy commutative property with respect to *, if ∀ a, b ∈ A, such that $a * b = b * a$

2. **Abelian Group: -** A non-empty set A is said to be a group with respect to a binary operation *, if A satisfy closure, Associative, identity, inverse, commutative property with respect to *.

		AS	SG	Monoid	Group	Abelian Group
1	(N, +)	Y	Y	N	N	
2	(N, -)	N	N	N	N	
3	(N, /)	N	N	N	N	
4	(N, x)	Y	Y	Y	N	
5	(Z, +)	Y	Y	Y	Y	
6	(Z, -)	Y	N	N	N	
7	(Z, /)	N	N	N	N	
8	(Z, x)	Y	Y	Y	N	
9	(R, +)	Y	Y	Y	Y	
10	(R, -)	Y	N	N	N	
11	(R, /)	N	N	N	N	
12	(R, x)	Y	Y	Y	N	
13	(M, +)	Y	Y	Y	Y	
14	(M, x)	Y	Y	Y	N	
15	(E, +)	Y	Y	Y	Y	
16	(E, x)	Y	Y	N	N	
17	(O, +)	N	N	N	N	
18	(O, x)	Y	Y	Y	N	
19	(R-0, x)	Y	Y	Y	Y	
20	(R-0, /)	Y	N	N	N	
21	(Non-Singular Matrix, x)	Y	Y	Y	Y	

Q Consider a set of natural numbers N, with respect to *, such that $a * b = a^b$ which of the following is true?

a) semi group but not monoid **b)** A monoid but not a group

c) A group **d)** not a semi group

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Q let {p, q, r, s} be the set. A binary operation * is defined on the set and is given by the following table:Which of the following is true about the binary operation?

*	p	q	r	s
P	p	r	s	p
q	p	q	r	s
r	p	q	p	r
S	p	q	q	q

a) it is commutative but not associative

b) it is associative but not commutative

c) it is both associative and commutative

d) it is neither associative nor commutative

Q Consider a set of integers Z, with respect to *, such that a * b = max (a, b) which of the following is true?

a) Algebraic structure

b) semi-group

c) Monoid

d) group

Q which of the following is not a group?

a) {……. -6, -4, -2, 0, 2, 4, 6, ……}, +

b) {…. -3k, -2k, -k, 0, k, 2k, 3k, ….}, + [k ∈ Z]

c) $\{2^n, n \in Z\}$, X

d) set of complex number, X

Q Consider the set of all integers(Z) with the operation defined as $m * n = m + n + 2$, $m, n \in Z$

if (z, *) forms a group, then determine the identity element

a) 0 **b)** -1 **c)** -2 **d)** 2

Q Consider a set of positive rational number with respect to an operation *, such that a*b = (aXb)/3, it is known that the it is an abelian group, which of the following is not true?

a) identity element e = 3

b) inverse of a = 9/a

c) inverse of 2/3 = 6

d) inverse of 3 = 3

- **Finite Group: -** A group with finite number of elements is called a finite group.

- **Order of group: -** Order of a group is denoted by O(G) = no of elements in G
  - If there is only one element in the Group, it must be an identity element.

Q Check out which of the following is a finite group?

1- {0}, +

+	0
0	

2- {1}, X

*	1
1	

3- {0,1}, +

+	0	1
0		
1		

4- {0,1}, X

*	0	1
0		
1		

5- {-1, 0, 1}, +

+	-1	0	1
-1			
0			
+1			

6- {-1, 0, 1}, X

*	-1	0	1
-1			
0			
1			

Q Check out which of the following is a finite group?

7- {-1, 1}, +

8- {-1, 1}, X

9- {-2, -1, 0, 1, 2}, +

+	-1	1
-1		
1		

*	-1	1
-1		
1		

+	-2	-1	0	1	2
-2					
-1					
0					
1					
2					

Q Check out which of the following is a finite group?

10- {-2, -1, 0, 1, 2}, X

*	-2	-1	0	1	2
-2					
-1					
0					
1					
2					

11- {1, ω, ω^2}, X

*	1	ω	ω^2
1			
ω			
ω^2			

12- {-1, 1, i, -i}, X

*	-1	1	i	-i
-1				
1				
i				
-i				

1. **Conclusion: -** it is very difficult to design finite group as with number greater than 2 closure property fails with simple addition and multiplication operation.

2. So we will try to develop new modified addition and multiplication operators with which closure and other properties can be satisfied.

- **Addition modulo: -** addition modulo is a binary operator denoted by $+_m$ such that

- $a +_m b = a + b \quad$ if $(a + b < m)$

- $a +_m b = a + b - m \quad$ if $(a + b >= m)$

$\{0,1,2,3\}, +_4$

$+_4$	0	1	2	3
0				
1				
2				
3				

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- **Multiplication modulo:** - Multiplication modulo is a binary operator denoted by $*_m$ such that

- $a \; X_m \; b = a \; X \; b$ if $(a \; X \; b < m)$

- $a \; X_m \; b = (a \; X \; b) \; \% \; m$ if $(a \; X \; b >= m)$

$\{1,2,3,4\}, X_5$

X_5	1	2	3	4
1				
2				
3				
4				

Q Check out which of the following is a group?

1- {0,1,2,3}, $+_4$

$+_4$	0	1	2	3
0				
1				
2				
3				

2- {0,1,2,3}, X_4

$*_4$	0	1	2	3
0				
1				
2				
3				

3- {1,2,3}, $+_4$

$+_4$	1	2	3
1			
2			
3			

4- {1,2,3}, X_4

$*_4$	1	2	3
1			
2			
3			

5- {0,1,2,3,4,5,6}, $+_7$

$+_7$	0	1	2	3	4	5	6
0							
1							
2							
3							
4							
5							
6							

6- {0,1,2,3,4,5,6}, X_7

$*_7$	0	1	2	3	4	5	6
0							
1							
2							
3							
4							
5							
6							

7- {1,2,3,4,5,6}, $+_7$

$+_7$	1	2	3	4	5	6
1						
2						
3						
4						
5						
6						

8- {1,2,3,4,5,6}, X_7

$*_7$	1	2	3	4	5	6
1						
2						
3						
4						
5						
6						

13- {1,3,5,7}, X_8 **14**- {1,2,4,7,8,11,13,14}, X_{15}

$*_8$	1	3	5	7
1				
3				
5				
7				

$*_{15}$	1	2	4	7	8	11	13	14
1								
2								
4								
7								
8								
11								
13								
14								

15- $\{1,2,3, 4......., p-1\}, X_p$

16- $\{0,1,2,3, 4......., p-1\}, X_p$

17- $\{1,2,3, 4......., p-1\}, +_p$

18- $\{0,1,2,3, 4......., p-1\}, +_p$

Q Consider the binary operation $\oplus$ over set $Z_n = \{0, 1, 2, \ldots\ldots, n-1\}$

$a \oplus b = a + b \quad \text{if } (a + b < n)$

$a \oplus b = a + b - n \quad \text{if } (a + b >= n)$

a) it is closed

b) it does not form a group

c) it forms a group but not an abelian group

d) it is an abelian group

Sub Group

1. The subset of a group may or may not be a group.

2. When the subset of a group is also a group then it is called sub group.

3. The identity element of a group and its sub group is always same.

4. Union of two subgroup may or may not be a subgroup.

5. Intersection of two subgroup is always a subgroup.

6. Lagrange's theorem: - the order of a group is always exactly divisible by the order of a sub group.

Q consider a group G = {1,3,5,7}, X_8 which of the following sub set of this set does not form is sub group?

a) {1,3} **b)** {1,5} **c)** {1,7} **d)** {1,3,7}

$*_8$	1	3
1		
3		

$*_8$	1	5
1		
5		

$*_8$	1	7
1		
7		

$*_8$	1	3	7
1			
3			
7			

Q Let G be a group with 15 elements. Let L be a subgroup of G. It is known that L != G and that the size of L is at least 4. The size of L is __________.

(A) 3 **(B)** 5 **(C)** 7 **(D)** 9

Q let (A, X) be a group of prime order, how many proper-subgroups are possible for A?

a) 0 **b)** 1 **c)** P-1 **d)** P

Order of an element: - (A, *) be a group, then ∀ a ∈ A, order of a is denoted by O(a).

1. O(a) =n (smallest positive integer), such that $a^n = e$

2. Order of identity element is always one.

3. Order of an element and its inverse is always same.

4. Order of an element in an infinite group does not exist or infinite expect identity.

5. Order of a group is always divisible by order of every element of the group.

Q consider a group {0,1,2,3}, $+_4$ and find the order of each element?

Q consider a set on cube root of unity $\{1, \omega, \omega^2\}$, X and find the order of each element?

Q consider a set on forth root of unity {-1, 1, I, -i}, X and find the order of each element?

<u>Generating element or Generator</u>: - A element 'a' is said to be a generating element, if every element of A is an integral power of a, i.e. every element of A can be represented using power of a.

$A = \{a^1, a^2, a^3, a^4, a^5 ....\}$

Cyclic group: - A group (A, *) is said to be a cyclic group if it contains at least one generator.

1. In a cyclic group if an element is a generator than its inverse will also be a generator.
2. The order of a cyclic group is always the order of the generating element of G.
3. Cyclic group is always alelian group.
4. Every group of order prime no is always always cyclic group where every number expect identity is generator.

Number of generators

Lagrange's theorem: - let A be a cyclic group of order n, number of Generator in A is denoted by $\phi(n) = \{n(p_1-1)(p_2-1)(p_3-1)\ldots\ldots\ldots(p_k-1)\} / (p_1p_2p_3\ldots\ldots\ldots p_k)$

Q let G be a cyclic group, O(G) = 8, number of generators in G =?

Q let G be a cyclic group, O(G) = 70, number of generators in G =?

Q Let s = set of all integers. A binary operation * is defined by

$a * b = a + b + 3$

consider the following statements

S_1: $(S,*)$ is a group

S_2: -3 is identity element of $(S, *)$

S_3: the inverse of -6 is 0

which of the following are true

a) Only S_1 and S_2

b) Only S_2 and S_3

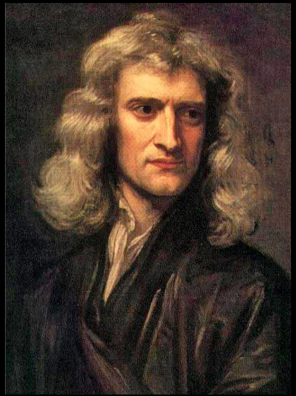
c) Only S_1 and S_3

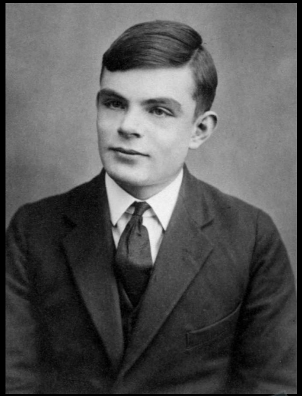
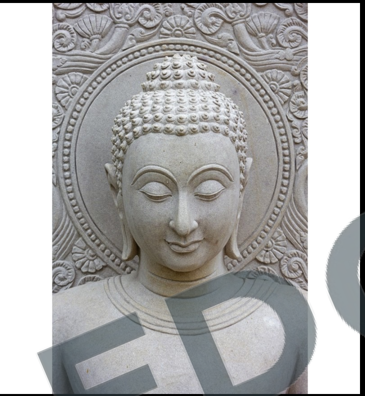
d) Only S_1, S_2 and S_3

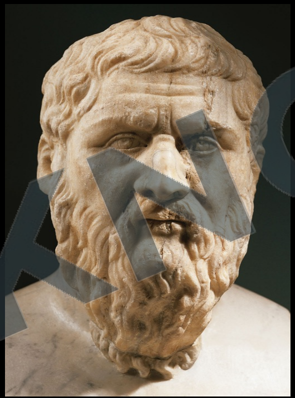
- **Chapter-7 (Proposition):**

Proposition

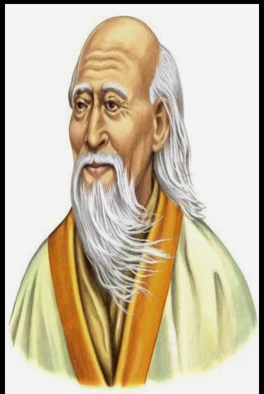
- First we must look at the difference between Scientist and Philosopher. Philosopher give an idea or theory which may have different interpretation from person to person. It depends on the wisdom of a person.



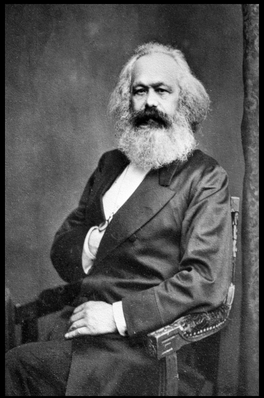



Confucius



Laozi




- **Logic in Reasoning:** Developed by Aristotle, it's a precise method for reasoning.
- **Beyond Propositions:** Other reasoning methods exist for problem-solving.
- **Role of Logic:** Central to mathematical statements, automated reasoning, and computer science.
- **Proofs and Theorems:** Correct arguments in math called proofs; proven statements are theorems.
- **Propositional Calculus:** A section of logic, also known as propositional or predicate logic, formalized by Aristotle.
- **Generating Propositions:** George Boole discussed methods in "The Laws of Thought" (1854).

- A **proposition** is a declarative sentence (that is, a sentence that declares a fact) that is either true or false, but not both.
  1. Delhi is the capital of USA
  2. How are you doing
  3. 5<= 11
  4. Temperature is less than 10 C
  5. It is cold today
  6. Read this carefully
  7. X + y = z

- Premises is a statement that provides reason or support for the conclusion(proposition). Premises(proposition) is always considered to be true.
- If a set of Premises(P) yield another proposition Q(Conclusion), then it is called an Argument.
- An argument is said to be valid if the conclusion Q can be derived from the premises by applying the rules of inference.

$\{P_1, P_2, P_3 \ldots, P_N\} \vdash Q$	P_1 P_2 P_3 . . . P_N …… Q …….	$\{P_1 \land P_2 \land P_3 \land \ldots . \land P_N\} \vdash Q$

- **Law of contradiction** **-** the law of non-contradiction (LNC) (also known as the law of contradiction, principle of non-contradiction (PNC), or the principle of contradiction) states that, ”Contradictory propositions cannot both be true in the same sense at the same time”.
    - e.g. the two propositions "*A is B*" and "*A is not B*" are mutually exclusive.

- **Law of excluded middle** **-** The law of excluded middle (or the principle of excluded middle) states that for any proposition, either that proposition is true or its negation is true.

Types of proposition

- We use letters to denote **propositional variables** to represent propositions, just as letters are used to denote numerical variables. The conventional letters used for propositional variables are *p, q, r, s*.

- Many mathematical statements are constructed by combining one or more propositions. New propositions, called **compound propositions**, are formed from existing propositions using logical operators.

Operators / Connectives

1. **Negation: -** let p be a proposition, then negation of p new proposition, denoted by $\neg p$, is the statement "it is not the case that p".

Negation	
P	¬P
F	
T	

Conjunction

- Let p and q be propositions. The *conjunction* of p and q, denoted by $p \wedge q$, is the proposition "p and q." The conjunction $p \wedge q$ is true when both p and q are true and is false otherwise.

Conjunction		
p	q	p ∧ q
F	F	
F	T	
T	F	
T	T	

Q Consider the following arguments and find which of them are valid?

1	
P_1	$(p \wedge q)$
Q	p

2	
P_1	P
Q	$p \wedge q$

3	
P_1	P
P_2	q
Q	$p \wedge q$

4	
P_1	$\neg(p \wedge q)$
P_2	P
Q	$\neg q$

5	
P_1	$\neg (p \wedge q)$
P_2	q
Q	$\neg p$

6	
P_1	$\neg (p \wedge q)$
P_2	$\neg p$
Q	q

7	
P_1	$\neg (p \wedge q)$
P_2	$\neg p$
Q	$\neg q$

Disjunction

- Let *p* and *q* be propositions. The *disjunction* of *p* and *q*, denoted by $p \vee q$, is the proposition. "*p* or *q*." The disjunction $p \vee q$ is false when both *p* and *q* are false and is true otherwise.

Disjunction		
p	q	p ∨ q
F	F	
F	T	
T	F	
T	T	

Q consider the following arguments and find which of them are valid?

1	
P_1	$(p \land q)$
Q	$p \lor q$

2	
P_1	$p \lor q$
Q	$(p \land q)$

3	
P_1	$\neg(p \lor q)$
Q	$\neg p$

4	
P_1	$(p \lor q)$
Q	$\neg p$

5	
P_1	$(p \lor q)$
P_2	$\neg p$
Q	q

6	
P_1	$(p \lor q)$
P_2	$\neg q$
Q	p

7	
P_1	$(p \lor q)$
P_2	p
Q	$\neg q$

8	
P_1	$(p \lor q)$
P_2	p
Q	q

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Implication

1. Let *p* and *q* be propositions. The *conditional statement* $p \rightarrow q$ is the proposition "if *p*, then q". The conditional statement $p \rightarrow q$ is false when *p* is true and *q* is false, and true otherwise.
2. In conditional statement $p \rightarrow q$, *p* is called the hypothesis (or antecedent or premise) and q is called the conclusion.

Implication		
p	q	$p \rightarrow q$
F	F	
F	T	
T	F	
T	T	

- Let p be the statement "Tori learns discrete mathematics" and q the statement "Tori will find a good job." Express the statement $p \rightarrow q$ as a statement in English.

1. "If Tori learns discrete mathematics, then she will find a good job."

2. "Tori will find a good job when she learns discrete mathematics."

3. "For Tori to get a good job, it is sufficient for her to learn discrete mathematics."

p	q	$P \to q$	$\neg p$	$\neg q$	$\neg P \to \neg q$	$q \to p$	$\neg q \to \neg p$	$\neg p \vee q$
F	F							
F	T							
T	F							
T	T							

- $p \rightarrow q$ *implication*
- $q \rightarrow p$ *converse*
- $\neg p \rightarrow \neg q$ *inverse*
- $\neg q \rightarrow \neg p$ *contra positive*
- $p \rightarrow q = \neg q \rightarrow \neg p$
- $p \rightarrow q$ *will be true if either p is false or q is true,* $p \rightarrow q = \neg p \lor q$

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Q Consider the following arguments and find which of them are valid?

Modus Ponens	
P_1	$p \to q$
P_2	P
Q	Q

Modus Tollens	
P_1	$p \to q$
P_2	$\neg Q$
Q	$\neg p$

1	
P_1	$\neg p$
Q	$p \to q$

2	
P_1	q
Q	$p \to q$

3	
P_1	$\neg(p \to q)$
Q	$\neg q$

4	
P_1	$\neg(p \to q)$
Q	p

Bi-conditional

- Let *p* and *q* be propositions. The bi*conditional statement p* $\leftrightarrow$ *q* is the proposition.
  - "*p if and only* q".
  - "*p* is necessary and sufficient for *q*"
  - "if *p* then *q*, and conversely"
  - "*p* iff *q*."
  - $p \leftrightarrow q = (p \rightarrow q) \wedge (q \rightarrow p)$
- The biconditional statement *p* $\leftrightarrow$ *q is true when p and q have the same values, and false otherwise.*

Bi-conditional		
p	q	$P \leftrightarrow q$
F	F	
F	T	
T	F	
T	T	

Q Consider the following arguments and find which of them are valid?

1	
P_1	$p \to q$
P_2	$q \to r$
Q	$p \to r$

2	
P_1	$p \vee q$
P_2	$p \to r$
P_3	$q \to r$
Q	r

3	
P_1	$p \vee q$
P_2	$p \to r$
P_3	$q \to s$
Q	$r \vee s$

4	
P_1	$p \to r$
P_2	$q \to s$
P_3	$\neg r \vee \neg s$
Q	$\neg p \vee \neg q$

Type of cases

- **Tautology/valid: -** A propositional function which is always having truth in the last column, is called tautology. E.g. $p \vee \neg p$

p	¬ p	p ∨ ¬ p
F	T	
T	F	

- **Contradiction/ Unsatisfiable: -** A propositional function which is always having false in the last column, is called Contradiction. E.g. $p \land \neg p$

p	¬ p	p ∧ ¬ p
F	T	
T	F	

- **Contingency: -** A propositional function which is neither a tautology nor a contradiction, is called Contingency. E.g. $p \vee q$

p	q	p V q
F	F	
F	T	
T	F	
T	T	

- **<u>Satisfiable</u>: -** A propositional function which is not contradiction is satisfiable. i.e. it must have at least one truth value in the final column e.g. $p \lor q$

- **Functionality Complete Set: -** A set of connectives is said to be functionally complete if it is able to write any propositional function.
  - $\{\wedge, \neg\}$
  - $\{\vee, \neg\}$

Q Consider two well-formed formulas in propositional logic

F_1: $P \Rightarrow \neg P$ **F_2**: $(P \Rightarrow \neg P) \vee (\neg P \Rightarrow P)$

Which one of the following statements is correct?

A) F_1 is satisfiable, F_2 is valid

B) F_1 unsatisfiable, F_2 is satisfiable

C) F_1 is unsatisfiable, F_2 is valid

D) F_1 and F_2 are both satisfiable

Q the statement $(\neg p) \implies (\neg q)$ is logically equivalent to which of the statement below?

1) $p \implies q$ **2)** $q \implies p$ **3)** $(\neg q) \vee (p)$ **4)** $(\neg p) \vee q$

a) 1 only **b)** 1 and 4 only **c)** 2 only **d)** 2 and 3 only

Q Consider the following two statements.

S_1: If a candidate is known to be corrupt, then he will not be elected.

S_2: If a candidate is kind, he will be elected.

Which one of the following statements follows from S1 and S2 as per sound inference rules of logic?

(A) If a person is known to be corrupt, he is kind

(B) If a person is not known to be corrupt, he is not kind

(C) If a person is kind, he is not known to be corrupt

(D) If a person is not kind, he is not known to be corrupt

Q Which one of the following is NOT equivalent to $p \leftrightarrow q$?

a) $(\neg p \lor q) \land (p \lor \neg q)$

b) $(\neg p \lor q) \land (q \rightarrow p)$

c) $(\neg p \land q) \lor (p \land \neg q)$

d) $(\neg p \land \neg q) \lor (p \land q)$

Q Consider the following statements:

P: Good mobile phones are not cheap

Q: Cheap mobile phones are not good

L: P implies Q

M: Q implies P

N: P is equivalent to Q

Which one of the following about L, M, and N is CORRECT?

(A) Only L is TRUE.

(B) Only M is TRUE.

(C) Only N is TRUE.

(D) L, M and N are TRUE

Q Which one of the following Boolean expressions is NOT a tautology?

A) $((a \to b) \wedge (b \to c)) \to (a \to c)$

B) $(a \to c) \to (\sim b \to (a \wedge c))$

C) $(a \wedge b \wedge c) \to (c \vee a)$

D) $a \to (b \to a)$

Q Consider the following logical inferences.

I_1: If it rains then the cricket match will not be played.
The cricket match was played.
Inference: There was no rain.

I_2: If it rains then the cricket match will not be played.
It did not rain.
Inference: The cricket match was played.

Which of the following is **TRUE**?

(A) Both I_1 and I_2 are correct inferences
(B) I_1 is correct but I_2 is not a correct inference
(C) I_1 is not correct but I_2 is a correct inference
(D) Both I_1 and I_2 are not correct inferences

Q Consider the following propositional statements:

P_1 : $((A \wedge B) \rightarrow C)) \equiv ((A \rightarrow C) \wedge (B \rightarrow C))$

P_2 : $((A \vee B) \rightarrow C)) \equiv ((A \rightarrow C) \vee (B \rightarrow C))$

Which one of the following is true?

(A) P_1 is a tautology, but not P_2

(B) P_2 is a tautology, but not P_1

(C) P_1 and P_2 are both tautologies

(D) Both P_1 and P_2 are not tautologies

Q Which of the following is/are a tautology?

a) $a \vee b \rightarrow b \wedge c$

b) $a \wedge b \rightarrow b \vee c$

c) $a \vee b \rightarrow (b \rightarrow c)$

d) $a \rightarrow b \rightarrow (b \rightarrow c)$

Q consider the following argument

I_1: if today is Gandhi ji's birthday, then today is oct 2^{nd}

I_2: today is oct 2^{nd}

C: today is Gandhi ji's birthday

Q consider the following argument

I_1: if Canada is a country, then London is a city

I_2: London is not a city

C: Canada is not a country

Q find which of the following arguments are valid?

1) $((p \vee q) \vee \neg p) = T$

2) $\neg (p \vee q) \vee (\neg p \wedge q) \vee p = T$

3) $((p \rightarrow q) \leftrightarrow (\neg q \rightarrow \neg p)) \wedge r = r$

4) $(p \lor q) \land \neg(\neg p \land (\neg q \lor \neg r)) \lor (\neg p \land \neg q) \lor (\neg p \land \neg r) = T$

5) $(p \lor \neg(p \land q)) = T$

6) $(p \land q) \land (\neg p \lor \neg q) = F$

7) $(\neg p \land (\neg q \land r)) \lor (q \land r) \lor (p \land r) = r$

First order Predicate Logic

- Sometime propositional logic cannot derive any meaningful information even though, we as human can understand that argument is meaningful or not.

- P_1: Every Indian like cricket
- P_2: Sunny is an Indian
- Q: Sunny Likes cricket

- The reason propositional logic fails here because using only inference system we can not conclude Q from P_1 and P_2.

- Sometime subject is not a single element but representing the entire group.
  - Every Indian like Cricket.
  - We can have a propositional function Cricket(x): x likes Cricket.
  - We can fix domain of discussion or universe of discourse as, x is an Indian.

- If i say four Indian are there I_1, I_2, I_3, I_4
- I_1 likes cricket ∧ I_2 likes cricket ∧ I_3 likes cricket ∧ I_4 likes cricket
- $Cricket(I_1) \land Cricket(I_2) \land Cricket(I_3) \land Cricket(I_4)$
- But problem with this notation is as there is 130+ corers Indian this formula will become very long and in some case we actually do not know how many subjects are there in the universe of discourse. so, we again need a short hand formula.
- $\forall_x$ Cricket(x), if we confine x to be Indian then it means every x like cricket.

- **Universal quantifiers**: - The universal quantification of a propositional function is the proposition that asserts

- P(x) is true for all values of x in the universe of discourse.

- The universe of discourse specifies the possible value of x.

- $\forall_x$ P(x), i.e. for all value of a P(x) is true

- Let try some other statement ‘Some Indian like samosa’
  - if i say four Indian are there I_1, I_2, I_3, I_4
  - I_1 like samosa $\vee$ I_2 like samosa $\vee$ I_3 like samosa $\vee$ I_4 like samosa
  - $Samosa(I_1) \vee Samosa(I_2) \vee Samosa(I_3) \vee Samosa(I_4)$
  - $\exists_x$ Samosa(x), if we confine x to be Indian then it means some x likes samosa.

- **Existential quantifiers**: - with existential quantifier of a propositional that is true if and only if P(x) is true for at least one value of x in the universe of discourse.

- There exists an element x is the universe of discourse such that P(x) is true.

- $\exists_x$ P(x), i.e. for at least one value of a P(x) is true

- let's change the universe of discourse from Indian to human
  - if human is Indian then it likes cricket
  - Indian(x): x is an Indian
  - Cricket(x): x likes Cricket

  - if I_1 is Indian then likes cricket ∧ if I_2 is Indian then likes cricket ∧ if I_3 is Indian then likes cricket ∧ if I_4 is Indian then likes cricket

  - $[\text{Indian}(I_1) \rightarrow \text{cricket}(I_1)] \land [\text{Indian}(I_2) \rightarrow \text{cricket}(I_2)] \land [\text{Indian}(I_3) \rightarrow \text{cricket}(I_3)] \land [\text{Indian}(I_4) \rightarrow \text{cricket}(I_4)]$

  - $\forall_x [\text{Indian}(x) \rightarrow \text{cricket}(x)]$

- let's change the universe of discourse from Indian to human
  - if human is Indian then it likes samosa
  - Indian(x): x is an Indian
  - Samosa(x): x likes Samosa

  - if I_1 is Indian then likes samosa $\vee$ if I_2 is Indian then likes samosa $\vee$ if I_3 is Indian then likes samosa $\vee$ if I_4 is Indian then likes samosa

  - $[\text{Indian}(I_1) \wedge \text{samosa}(I_1)] \vee [\text{Indian}(I_2) \wedge \text{samosa}(I_2)] \vee [\text{Indian}(I_3) \wedge \text{samosa}(I_3)] \vee [\text{Indian}(I_4) \wedge \text{samosa}(I_4)]$

  - $\exists_x [\text{Indian}(x) \wedge \text{samosa}(x)]$

Negation

- $\neg\ [\forall_x\ P(x)] = \exists_x\ \neg P(x)$

- $\neg\ [\exists_x\ P(x)] = \forall_x\ \neg P(x)$

Let L(x, y): x like y, which means x likes y or y is liked by x

1- $\forall_x \forall_y L(x, y)$

2- $\forall_y \forall_x L(x, y)$

3- $\exists_x \exists_y L(x, y)$

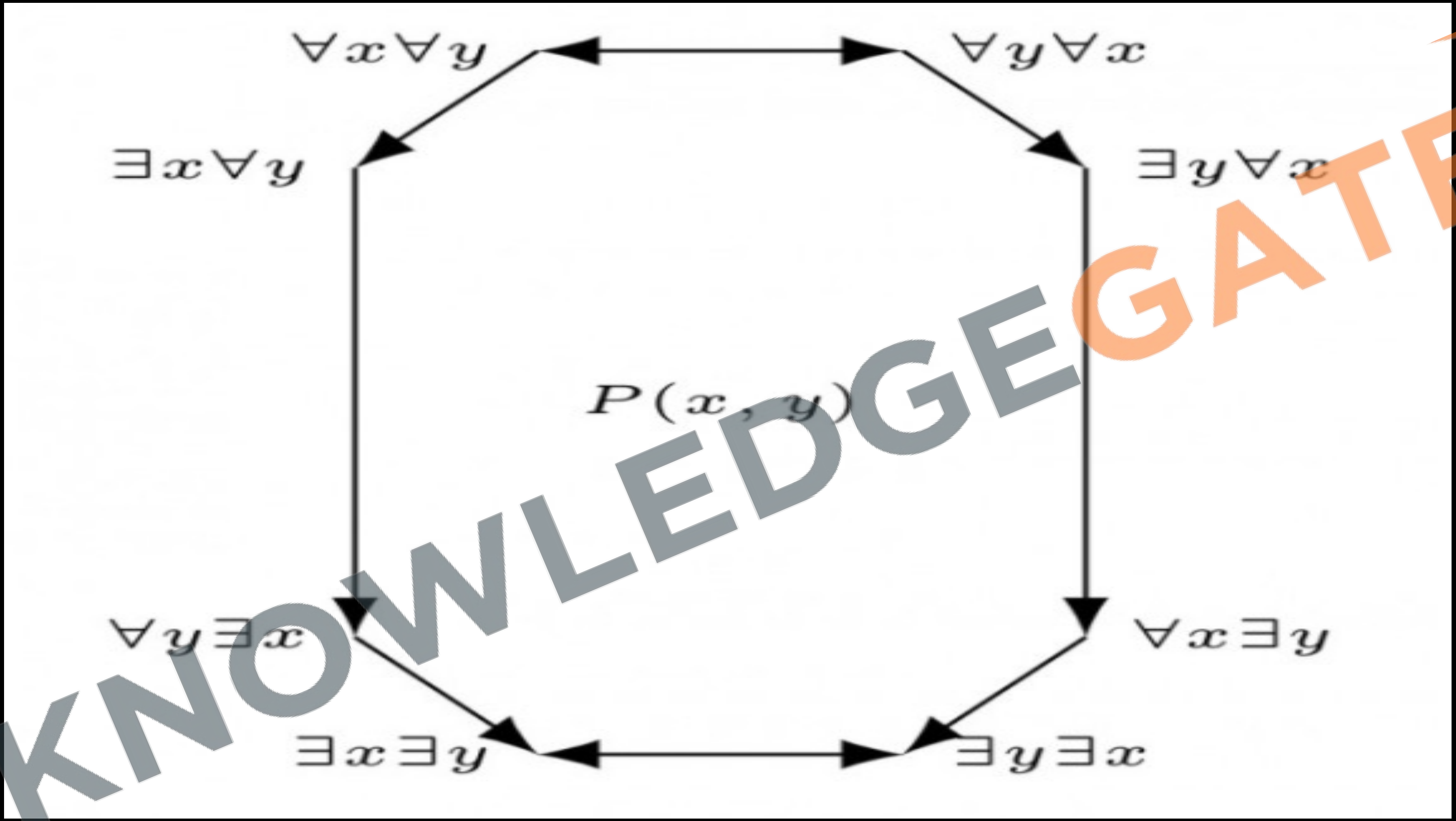
4- $\exists_y \exists_x L(x, y)$

5- $\forall_x \exists_y L(x, y)$

6- $\exists_y \forall_x L(x, y)$

7- $\forall_y \exists_x L(x, y)$

8- $\exists_x \forall_y L(x, y)$


∀x∀y
∀y∀x
∃x∀y
∃y∀x
P(x, y)
∀y∃x
∀x∃y
∃x∃y
∃y∃x

	1
P_1	$\exists_x P(x) \vee \exists_x Q(x)$
Q	$\exists_x (P(x) \vee Q(x))$

	2
P_1	$\exists_x (P(x) \vee Q(x))$
Q	$\exists_x P(x) \vee \exists_x Q(x)$

	3
P_1	$\exists_x P(x) \wedge \exists_x Q(x)$
Q	$\exists_x (P(x) \wedge Q(x))$

	4
P_1	$\exists_x (P(x) \wedge Q(x))$
Q	$\exists_x P(x) \wedge \exists_x Q(x)$

	1
P_1	$\forall_x P(x) \vee \forall_x Q(x)$
Q	$\forall_x (P(x) \vee Q(x))$

	2
P_1	$\forall_x (P(x) \vee Q(x))$
Q	$\forall_x P(x) \vee \forall_x Q(x)$

	3
P_1	$\forall_x P(x) \wedge \forall_x Q(x)$
Q	$\forall_x (P(x) \wedge Q(x))$

	4
P_1	$\forall_x (P(x) \wedge Q(x))$
Q	$\forall_x P(x) \wedge \forall_x Q(x)$

Q consider the statement $\exists_x[P(x) \wedge \neg Q(x)]$, Which of the following is equivalent?

a) $\forall_x [P(x) \rightarrow Q(x)]$

b) $\forall_x [\neg P(x) \rightarrow Q(x)]$

c) $\neg \{\forall_x [P(x) \rightarrow Q(x)]\}$

d) $\neg \{\forall_x [\neg P(x) \rightarrow Q(x)]\}$

Q negation of the statement

$\exists_x \forall_y [F(x, y) \rightarrow \{G(x, y) \lor H(x, y)\}] = \forall_x \exists_y [F(x, y) \land \{\neg G(x, y) \land \neg H(x, y)\}]$?

Q let in a set of all integers

G (x, y): x is greater than y

"for any given positive integer, there is a greater positive integer"

a) $\forall_x \exists_y G(x, y)$

b) $\exists_y \forall_x G(x, y)$

c) $\forall_y \exists_x G(x, y)$

d) $\exists_x \forall_y G(x, y)$

Q let in a set of all humans

L (x, y): x likes y

“there is someone, whom no one like”

a) $\forall_x \exists_y \{\neg L(x, y)\}$

b) $\{\neg \forall_x \exists_y L(x, y)\}$

c) $\neg \{\forall_y \exists_x L(x, y)\}$

d) $\neg \{\exists_y \forall_x L(x, y)\}$

Q The CORRECT formula for the sentence, **"not all rainy days are cold"** is **(GATE-2014) (2 Marks)**

a) $\forall_d$ (Rainy(d) $\land$ ~Cold(d))

b) $\forall_d$ (~Rainy(d) $\rightarrow$ Cold(d))

c) $\exists_d$ (~Rainy(d) $\rightarrow$ Cold(d))

d) $\exists_d$ (Rainy(d) $\land$ ~Cold(d))

Q What is the logical translation of the following lowing statement? **(GATE-2013) (2 Marks)**

"None of my friends are perfect."

A) $\exists_x(F(x) \land \neg P(x))$

B) $\exists_x(\neg F(x) \land P(x))$

C) $\exists_x(\neg F(x) \land \neg P(x))$

D) $\neg\exists_x(F(x) \land P(x))$

Q Which one of the following options is CORRECT given three positive integers x, y and z, and a predicate? **(GATE-2011) (2 Marks)**

$$P(x) = \neg(x=1) \wedge \forall_y(\exists_z(x=y*z) \Rightarrow (y=x) \vee (y=1))$$

(A) P(x) being true means that x is a prime number

(B) P(x) being true means that x is a number other than 1

(C) P(x) is always true irrespective of the value of x

(D) P(x) being true means that x has exactly two factors other than 1 and x

Q Consider the following well-formed formulae:

1) $\neg\forall x(P(x))$ **2)** $\neg\exists x(P(x))$ **3)** $\neg\exists x(\neg P(x))$ **4)** $\exists x(\neg P(x))$

Which of the above are equivalent? **(GATE-2009) (2 Marks)**

a) I and III

b) I and IV

c) II and III

d) II and IV

Q Which one of the following is the most appropriate logical formula to represent the statement? **"Gold and silver ornaments are precious".** The following notations are used:

G(x): x is a gold ornament

S(x): x is a silver ornament

P(x): x is precious **(GATE-2009) (2 Marks)**

(A) $\forall_x (P(x) \rightarrow (G(x) \wedge S(x)))$

(B) $\forall_x ((G(x) \wedge S(x)) \rightarrow P(x))$

(C) $\exists_x ((G(x) \wedge S(x)) \rightarrow P(x)$

(D) $\forall_x ((G(x) \vee S(x)) \rightarrow P(x))$